REDUCED-ORDER MODELING OF NEUTRON TRANSPORT SEPARATED IN SPACE AND ANGLE VIA PROPER GENERALIZED DECOMPOSITION

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ABSTRACT
As radiation transport problems are posed in a high-dimensional phase space (of position \( r = (x, y, z) \), angle \( \Omega = (\theta, \varphi) \), energy \( E \), and time \( t \)), seeking a Reduced-Order Model (ROM) via separated representations may prove advantageous. Proper Generalized Decomposition (PGD), as an \textit{a priori} ROM strategy, is uniquely attractive, given it seeks this separated representation iteratively, eschewing the need for “snapshots” of the full-order solution (as in \textit{a posteriori} ROM methods). We here pursue a method for first-order neutron transport in slab geometry, using PGD to separate space and angle. This method elaborates on the work of [1] by introducing bidirectional transport and that of [2] by considering collision and scattering operators. Verification of the expected order of accuracy is demonstrated via the Method of Manufactured Solutions (MMS) while verification with enrichment (increasing number of PGD iterations) proceeds by way of a semi-analytic benchmark. Lastly, a parametric study over the scattering ratio is conducted with asymmetric, isotropic boundary conditions. In all cases, the proposed PGD representation is found to be appropriate and convergent with a tractable number of iterations.

KEYWORDS: Proper Generalized Decomposition, Reduced Order Modeling, deterministic transport

1. INTRODUCTION
Given radiation transport problems are inherently high-dimensional, the computational cost scales rapidly as \( O(N_r N_\Omega N_E N_t) \) where space \( r = (x, y, z) \), angle \( \Omega = (\theta, \mu) \), energy \( E \), and time \( t \). This burden can render simulations of even moderate refinement costly or intractable, necessitating physical simplifications of the model or restriction of the problem domain. By contrast, Reduced-Order Models (ROMs) offer a model which retains the features of the original, but at a reduced computational effort, disrupting this scaling law. Proper Generalized Decomposition (PGD), as an \textit{a priori} ROM strategy, is particularly useful, as it seeks this separated representation iteratively. This is in contrast to \textit{a posteriori} ROM methods which require multiple realizations, or “snapshots”, of the full-order solution to construct the ROM. The advantage here is appreciable, as computation and storage of these snapshots may require immense (potentially prohibitive) processing and memory resources in practical neutron transport applications.

Prior work applying PGD for neutron transport exists but has largely been restricted to diffusion theory, beginning with González-Pintor et al. [3] who utilize a 2D spatially decomposed one-group
diffusion method to compute the multiplication factor of a nuclear reactor. Likewise, Senecal and Ji [4] offer a similar 2D spatially decomposed diffusion method extended to two neutron energy groups. Lastly, Alberti and Palmer pursue a transient 1D diffusion method, using PGD to separate space and time, most recently demonstrated in [5].

Regarding PGD for Boltzmann transport generally, Chinesta et al. [2] present results separated in space, velocity, and time (for 1D and 2D geometries, as well as for steady-state scenarios), yet omit phenomena specific to neutron transport (namely, collision and scattering). Most recently, Dominesey, Senecal, and Ji [1] explore Boltzmann transport for neutrons specifically (decomposed in space and angle via PGD) and discuss practicalities including source iteration and Anderson acceleration for fixed-point iterations. However, this effort considers monodirectional transport only, leaving a bidirectional implementation as future work.

As such, we herein pursue a bidirectional method for first-order neutron transport in slab geometry, using PGD to separate space and angle. This method expands on the work of [1] by introducing bidirectional transport and that of [2] by considering collision and scattering operators particular to neutron transport. The expected order of accuracy is verified by the Method of Manufactured Solutions (MMS) while convergence with enrichment is verified using a semi-analytical benchmark by Ganapol [6]. Finally, numerical results over variable scattering ratios are presented for a representative problem with asymmetric, isotropic boundary conditions.

2. THEORY

The central axiom of PGD is that a solution \( u \) may be approximated via a separated representation

\[
u(x_1, \ldots, x_D) \approx \sum_{m=1}^{M} \prod_{d=1}^{D} X^d_m(x_d)
\]

where each subscript or superscript \( d \) denotes a unique dimension of \( u \). At present, we seek the angular flux \( \psi(x, \mu) \) in terms of spatial and angular modes, \( X_m(x) \) and \( U_m(\mu) \), such that

\[
\psi(x, \mu) \approx \sum_{m=1}^{M} X_m(x)U_m(\mu).
\]

2.1. Application to Neutron Transport

We begin with the monoenergetic neutron transport equation in slab geometry with isotropic scattering

\[
\frac{\partial \psi}{\partial x}(x, \mu) + \Sigma_t(x)\psi(x, \mu) = \Sigma_s(x) \int_{-1}^{1} \frac{1}{2} \psi(x, \mu')d\mu' + Q(x, \mu)
\]

with the boundary conditions

\[
\psi(x, \mu) = \begin{cases} 
\psi_l(\mu), & x = 0, \mu > 0 \\
\psi_r(\mu), & x = L, \mu < 0
\end{cases}
\]
where \( L \) is the length of the slab, and \( \psi_L \) and \( \psi_R \) are known functions representing the incident flux on the left and right edges of the slab. We obtain the weak form by multiplying by an arbitrary test function \( \varphi^*(x, \mu) \) and integrating over all \( x \in \Omega_x \) and \( \mu \in \Omega_\mu \). For brevity, let us introduce the notation
\[
(\bullet, \bullet)_{\Omega_{D_1} \times \Omega_{D_2} \ldots} = \int_{\Omega_{D_1} \times \Omega_{D_2} \ldots} \bullet \bullet \ dD_1 dD_2 \ldots
\] (5)
such that we find
\[
\left( \mu \frac{\partial \psi}{\partial x}, \varphi^* \right)_{\Omega_x \times \Omega_\mu} + (\Sigma_t \psi, \varphi^*)_{\Omega_x \times \Omega_\mu} - \left( \Sigma_s \int_{-1}^1 \frac{1}{2} \psi d\mu', \varphi^* \right)_{\Omega_x \times \Omega_\mu} = (Q, \varphi^*)_{\Omega_x \times \Omega_\mu}
\] (6)
where we have omitted all functional dependencies on \( x \) and \( \mu \).

2.2. Bidirectional Separation

However, we now observe that enforcing our boundary conditions would be infeasible over \( \Omega_\mu = [-1, 1] \), so long as \( X_m \) is independent of \( \mu \). As such, we restrict \( \mu \) to \( \Omega_\mu = (0, 1] \) and introduce a variable \( v \) to represent the direction of particle motion, where \( \Omega_v = \{-1, 1\} \) as in [2]. We then redefine our spatial modes as
\[
X_m(x, v) = \begin{cases} 
X^+(x), & v = +1 \\
X^-(x), & v = -1 
\end{cases}
\] (7)
allowing us to write an alternative but equivalent weak form
\[
\left( v \mu \frac{\partial \psi}{\partial x}, \varphi^* \right)_{\Omega_x \times \Omega_\mu \times \Omega_v} + (\Sigma_t \psi, \varphi^*)_{\Omega_x \times \Omega_\mu \times \Omega_v} - \left( \Sigma_s \int_{\Omega_v} \int_{0}^{1} \frac{1}{2} \psi dv' d\mu', \varphi^* \right)_{\Omega_x \times \Omega_\mu \times \Omega_v} = (Q, \varphi^*)_{\Omega_x \times \Omega_\mu \times \Omega_v}
\] (8)
on which we can feasibly enforce boundary conditions (as visualized in Figure 1).

In keeping with PGD methodology, we assume a separable form of our test function,
\[
\varphi^*(x, \mu) = X^*(x, v)U_M(\mu) + X_M(x, v)U^*(\mu),
\] (9)
where \( X_M \) and \( U_M \) represent the unknown \( M^{th} \) modes of the approximate angular flux, and \( X^* \) and \( U^* \) are arbitrary functions. Similar to \( X_m \), we also assume our arbitrary \( X^*(x, v) \) function may be defined in a piecewise manner as \( X^*_\pm(x) \) for \( v = \pm 1 \). Moreover, we take our source to be separable as
\[
Q(x, \mu) = \sum_{n=1}^{N} Q^{(n)}_x(x)Q^{(n)}_\mu(\mu).
\] (10)
Note that all source functions \( Q(x, \mu) \) will be discrete in practice, such that a factorization of this form can be easily achieved (for example, by the Singular Value Decomposition, or SVD) even if the continuous source is not given in a separated form.

After substituting our separable test function and flux approximation into our alternative weak form, we invoke the arbitrariness of \( X^* \) to \( U^* \) to yield one equation for each term of the test
function. As \( v \) is discrete-valued, we note each integral over \( \Omega_v \) is more appropriately described as a summation for \( v = \pm 1 \). Carrying out this summation we find for \( X_M^* U_M \),

\[
\sum_{m=1}^{M} \left( \frac{dX_m^\pm}{dx}, X_m^\pm \right)_{\Omega_x} (\mu U_m, U_M)_{\Omega_\mu} + \sum_{m=1}^{M} (\Sigma_t X_m^\pm, X_m^\pm)_{\Omega_x} (U_m, U_M)_{\Omega_\mu} - \sum_{m=1}^{M} \left( \Sigma_s X_m^\pm, X_m^\pm \right)_{\Omega_x} \left( \int_0^1 \frac{1}{2} U_m d\mu', U_M \right)_{\Omega_\mu} = \sum_{n=1}^{N} (Q_{x,(n)}^+, X_m^\pm)_{\Omega_x} (Q_{\mu,(n)}^+, U_M)_{\Omega_\mu} + \sum_{m=1}^{M} \left( \Sigma_s X_m^\pm, X_m^\pm \right)_{\Omega_x} \left( \int_0^1 \frac{1}{2} U_m d\mu', U_M \right)_{\Omega_\mu}
\]

and for \( X_M U^* \),

\[
\sum_{m=1}^{M} \left[ \left( \frac{\partial X_m^+}{\partial x}, X_M^+ \right)_{\Omega_x} - \left( \frac{\partial X_M^-}{\partial x}, X_M^- \right)_{\Omega_x} \right] (\mu U_m, U^*)_{\Omega_\mu} + \sum_{m=1}^{M} \left[ \Sigma_t X_M^+, X_M^+ \right)_{\Omega_x} + \left( \Sigma_t X_M^-, X_M^- \right)_{\Omega_x} (U_m, U^*)_{\Omega_\mu} - \sum_{m=1}^{M} \left[ \Sigma_s (X_M^+ + X_m^+), X_M^+ \right)_{\Omega_x} + \left( \Sigma_s (X_M^- + X_m^-), X_M^- \right)_{\Omega_x} \left( \int_0^1 \frac{1}{2} U_m d\mu', U^* \right)_{\Omega_\mu} = \sum_{n=1}^{N} \left[ (Q_{x,(n)}^+, X_M^+) \right)_{\Omega_x} + \left( Q_{\mu,(n)}^+, X_M^+ \right)_{\Omega_x} (Q_{\mu,(n)}^+, U_M)_{\Omega_\mu}.
\]

Together, these three equations represent a nonlinear problem, as each equation contains products of unknowns (namely, \( X_M^\pm \) and \( U_M \)). Accordingly, we must employ a suitable linearization strategy, iterating between Equations 11 and 12 to solve this system. In doing so, each underlined term coupling these equations becomes an iterative constant.

![Diagram](a) Full-order problem domain (b) PGD problem domain

\textbf{Figure 1: Schematic of the bidirectional problem domain.}

Moreover, while the coupling by scattering between \( X_M^+ \) and \( X_M^- \) (as represented in the final term of Equation 11) is linear, we nevertheless choose to iterate between these two directional modes.
Doing so avoids doubling the size of our linear system, at the cost of some number of “source iterations” (as they are commonly referred to in $S_N$ methods) by which we must iterate on the scattering source. Additionally, while in slab geometry this mitigates only a doubling of the linear system, in 2D and 3D simultaneously solving along all directions would quadruple and octuple the size of the linear system, further motivating an iterative scheme.

2.3. Picard Iteration

Ultimately, Equations 11 and 12 present a nonlinear problem which may be solved to find the latest modes $X_M^+, X_M^-, \text{ and } U_M$, given a suitable linearization strategy. In what follows, we employ Picard (or fixed-point) iteration to solve this nonlinear system. Specifically, we begin by guessing $U_M^{(0)}$, $X_M^{+, (0)}$, and $X_M^{-(0)}$, subsequently solve for $X_M^{+, (1)}$ and $X_M^{-(1)}$, and then solve for $U_M^{(1)}$. This alternating iteration (omitting the initial guess) is repeated until some error indicator, here taken to be a relative $L^2$ measure

$$\varepsilon_k = \frac{\| \left( X_M^{+, (k)} + X_M^{-(k)} \right) U_M^{(k)} - \left( X_M^{+, (k-1)} + X_M^{-(k-1)} \right) U_M^{(k-1)} \|_2}{\| \left( X_M^{+, (k-1)} + X_M^{-(k-1)} \right) U_M^{(k-1)} \|_2}$$

falls below a user-prescribed tolerance, where $k$ denotes the current fixed-point iteration. Note also that we normalize $X_M^{+, (k)}$ and $X_M^{-(k)}$ within each Picard iteration as

$$X_M^{\pm, (k)} := X_M^{\pm, (k)} / \left( \| X_M^{+, (k)} \|_2 + \| X_M^{-, (k)} \|_2 \right)$$

where $\| \cdot \|_2$ represents the vector $L^2$ norm. This normalization mitigates the “drifting” of modes between iterations, wherein one mode (either $X_M$ or $U_M$) grows while the other shrinks. We summarize our PGD procedure (with slight modification of nomenclature) in Algorithm 1.

3. NUMERICAL RESULTS

3.1. Discretization

At present, we revert to the strong form of our two advection equations for the spatial modes and discretize using upwind finite differences (that is, backward Euler for $X_M^+$ and forward Euler for $X_M^-$). Accordingly, we achieve $O(h)$ numerical accuracy, where $h$ is conventionally the “mesh width”. Note that any suitable scheme, including finite element or finite volume methods, could be used instead. However, in solving advective equations, appropriate upwinding of the discretization is necessary to avoid spurious oscillations in space. This motivates our selection of finite differences, as upwinding is straightforward both in theory and implementation.

Meanwhile, for $U_M$ in angle, we have an integral equation (specifically, a Fredholm integral equation of the second kind) which can be numerically solved via any appropriate quadrature method. At present, we revert to a strong form of the angular equation and select a trapezoidal approximation of the integral scattering operator with order of accuracy $O(h^2)$. Note that despite this second-order accuracy, convergence of the method as a whole will be limited by the $O(h)$ convergence of our spatial discretization, as we will subsequently demonstrate and verify using MMS.
Algorithm 1: Space-Angle PGD

\[
\begin{align*}
\psi_+ & \leftarrow 0; \quad \text{// Let } \psi_+ = \psi(x, +\mu) \\
\psi_- & \leftarrow 0; \quad \text{// Let } \psi_- = \psi(x, -\mu)
\end{align*}
\]

for \( m \leftarrow 1, 2 \) do Boundary Conditions
  Prescribe \( X_+^m, X_-^m \) and \( U_m \) to satisfy boundary conditions;
  \[
  \begin{align*}
  \psi_+ & \leftarrow \psi_+ + X_+^m \otimes U_m; \quad \text{// \( \otimes \) denotes the tensor product} \\
  \psi_- & \leftarrow \psi_- + X_-^m \otimes U_m;
  \end{align*}
  \]
  // Can be omitted for homogenous Dirichlet boundary conditions.
  // Only \( m = 1 \) required if \( \psi_\ell \) and \( \psi_r \) vary equivalently in angle.

for \( m \leftarrow 3, 4 \ldots M \) do Enrichment Loop
  Initialize \( X_+^{m,(0)}, X_-^{m,(0)}, \) and \( U_m^{(0)} \) as guesses; \quad // Here taken to be uniform
  for \( k \leftarrow 1, 2 \ldots \) do Picard Loop
    \[
    \begin{align*}
    X_+^{m,(k)} & \leftarrow \text{solution of Equation 11 given } X_-^{m,(k-1)} \text{ and } U_m^{(k-1)}; \\
    X_-^{m,(k)} & \leftarrow \text{solution of Equation 11 given } X_+^{m,(k-1)} \text{ and } U_m^{(k-1)}; \\
    X_m^{\pm,(k)} & \leftarrow X_m^{\pm,(k)}/\left(||X_m^{(k)}||_2 + ||X_m^{(k)}||_2\right) \quad \text{// Normalization} \\
    U_m^{(k-1)} & \leftarrow \text{solution of Equation 12 given } X_+^{m,(k)} \text{ and } X_-^{m,(k)}; \\
    \varepsilon_k & \leftarrow \text{compute by Equation 13}; \quad \text{// Here taken to be } 10^{-6} \text{ by default}
    \end{align*}
    \]
    if \( \varepsilon_k < \text{tolerance} \) then
      \[
      \begin{align*}
      X_m^\pm & \leftarrow X_m^{\pm,(k)}; \\
      U_m & \leftarrow U_m^{(k)}; \\
      \text{break}
      \end{align*}
      \]
  \[
  \begin{align*}
  \psi_+ & \leftarrow \psi_+ + X_+^m \otimes U_m; \\
  \psi_- & \leftarrow \psi_- + X_-^m \otimes U_m;
  \end{align*}
  \]

3.2. Verification by Method of Manufactured Solutions

By MMS, let us simulate a problem in a two centimeter slab with a known solution
\[
\begin{align*}
\psi(x, \mu, v = +1) & = \sin(\pi x/2) (1 - \cos(\pi \mu)) \\
\psi(x, \mu, v = -1) & = \sin(\pi x/2) (1 - \cos(\pi \mu)) \cdot 0.5.
\end{align*}
\] (15)

and cross-sections \( \Sigma_t = 1 \), \( \Sigma_s = 0.9 \) using \( N \) mesh nodes in both \( x \) and \( \mu \). Measuring the convergence of the method by a relative \( L^2 \) error indicator
\[
L^2 \text{ Error} = ||\psi - \psi_{\text{exact}}||_2/||\psi_{\text{exact}}||_2
\] (16)
and truncating the PGD approximation at \( M = 1 \) we achieve Figure 2 which demonstrates the expected \( O(h) \) numerical accuracy (where \( h \) is inversely proportional to \( N - 1 \)).

3.3. Verification by Analytical Benchmark

Having demonstrated the expected order of accuracy using MMS, we next seek to verify our method with increasing enrichment (that is, number of modes \( M \)) against an analytical benchmark of prototypical importance to neutron transport. Specifically, we compare against the semi-analytic
benchmark of Ganapol [6] which describes a slab impinged on by an incident flux at the left and
with a vacuum boundary, \( \psi_r(\mu) = 0 \), on the right, with isotropic scattering and no loss of energy.

More specifically, we take the incident flux \( \psi_\ell(\mu) \) to be a monodirectional unit source normal to
the slab (nonzero only for \( \mu = 1 \)) and the slab to be one mean-free-path thick with a scattering
ratio \( c \) of 0.9 where \( c = \Sigma_s/\Sigma_t \). However, as our implementation is discrete, a true normal beam
cannot be represented exactly. Instead, we impose a vacuum boundary condition on the left and
represent the normal beam as an effective (once-collided) scattering source \( \tilde{Q} \), namely

\[
\tilde{Q}(x, \mu) = \Sigma_s(x) \int_0^1 \frac{1}{2} \tilde{\psi}(x, \mu') d\mu'
\]

where \( \tilde{\psi}(x, \mu') \) is the uncollided flux arising from the left boundary, \( \tilde{\psi}(0, \mu') = \psi_\ell(\mu') \). Analyti-
cally, by the Beer–Lambert law,

\[
\tilde{\psi}(x, \mu') = \psi_\ell(\mu') e^{-\int_0^s \Sigma_t(x) ds}
\]

where \( s \) is the path length, \( s = x/\mu' \). We here substitute \( \psi_\ell(\mu') = \delta(\mu' - 1) \) to yield

\[
\tilde{Q}(x, \mu) = \tilde{Q}(x) = \frac{1}{2} \Sigma_s(x) e^{-\int_0^s \Sigma_t(x) dx}
\]

allowing us to compute the (once or more) collided flux for increasing number of modes \( M = 5, 10, 15, 20 \) as in Figure 3, having discretized using 300 nodes in space and angle.

From these plots we generally observe considerable agreement between the analytical and nu-
merical PGD fluxes for \( M \geq 10 \). However, the solution near \( \mu = 0 \) for both the transmitted
and reflected fluxes is apparently deficient. One may hypothesize this to be a consequence of
the discontinuity at \( \psi(0, 0) \), as \( \lim_{x \to 0^+} \psi(x, \mu) = \psi_\ell(\mu) \) except as \( \mu \to 0^+ \). Naturally, this jump
discontinuity may render the solution difficult to approximate in a separated fashion along \( \mu = 0 \).

Future efforts will seek to compare against such benchmarks globally (as opposed to only at the
slab edges) and investigate the effects of alternative boundary fluxes, \( \psi_\ell(\mu) \). The effects of differing
PGD algorithms and numerical discretizations may also be explored in further works.

**Figure 2: Verification of first-order numerical accuracy for \( M = 1 \) by MMS.**
3.4. Study over Scattering Ratio

We next move to simulate a problem lacking a known, analytical solution. For ease of demonstration, we employ isotropic Dirichlet boundary conditions of unit strength on the left edge of the slab and one-half strength on the right.

To illustrate the effects of scattering, we simulate this model over two mean-free-paths for scattering ratios $c = 0.5$, 0.9, and 1.0 and $\Sigma_t(x) = 1.0 \text{ cm}^{-1}$. Spatially-dependent cross-sections (that is, transport through heterogeneous media) can be considered by the method (and have been implemented), but are not presented here for simplicity. We simulate bidirectional transport through a two centimeter slab with no internal source $Q(x,\mu)$ using 200 spatial nodes, 200 angular nodes, and a Picard tolerance of $10^{-4}$ for 20 computed modes. To enforce boundary conditions, the first mode is assumed to be

$$X_1^+ = \begin{cases} 1, & x = 0, \\ 0, & x > 0, \end{cases} \quad X_1^- = \begin{cases} 0.5, & x = 2, \\ 0, & x < 2, \end{cases} \quad U_1(\mu) = 1,$$ (20)

and all subsequent modes are required to vanish on the boundaries (as is typical for PGD methods).

As the convergence of fixed-point iterations is not guaranteed, we here impose a maximum number of 99 Picard iterations. Note this limit is intentionally high, as we aim to ascertain the convergence of the PGD enrichment without complications of fixed-point error, to which end we label the modes $m$ of Figure 4 where we reach this threshold. In practice, coarse fixed-point error indicators and low iteration limits are often advisable, as any remaining fixed-point error can be corrected by later modes (or minimized by “projection” or “update” of the PGD reduced basis).

In measuring the convergence with enrichment (increasing $M$) Figure 4 employs three indicators. Firstly the relative $L^2$ norm, $||U_m||_2/||U_2||_2$ describes the importance of a given angular mode $U_m$ compared to the first computed mode $U_2$. As both spatial modes $X_1^+$ and $X_1^-$ are normalized, it is sufficient to measure only the angular modes. For a more physically intuitive metric, $\max(\Delta\psi_m)$ or $\max(||X_m^+ + X_m^-||_\infty \cdot ||U_m||_\infty)$ provides the maximum change in angular flux $\psi$ induced by mode $m$. Likewise, $\max(\Delta\phi_m)$ or $||X^+_m + X^-_m||_\infty \cdot \int_0^1 U_m d\mu'$ describes the analogous indicator for the scalar (angle-integrated) flux. For reference, scalar fluxes are displayed in Figure 5.
From these plots we observe that the relative $L^2$ significance of the final angular modes is approximately $2 \times 10^{-3}$ or less that of the first computed mode, $U_2$. The maximum absolute local changes in angular flux, meanwhile, are on the order of $10^{-2}$ [cm$^{-2}$s$^{-1}$] or less while that of the scalar flux are found to be in the vicinity of $10^{-3}$ [cm$^{-2}$s$^{-1}$]. Crucially, we find that varying the scattering ratio does not appear to have an adverse impact on the PGD approximation. Moreover, despite
restricting $\mu$ to $(0, 1]$ to compute a directionally-independent $U(\mu)$, the present method appears capable of adequately representing fluxes with dissimilar directional angular distributions (including, as in Figure 4c, effectively opposite distributions).

4. CONCLUSIONS

In conclusion, we have derived and demonstrated a method for bidirectional first-order neutron transport in slab geometry which employs PGD to separate the spatial and angular dimensions. Discretization proceeds via upwind finite differences in space and a trapezoidal quadrature rule in angle. $O(h)$ numerical accuracy is verified by MMS while a semi-analytical benchmark is employed for verification with enrichment in practical scenarios. A parametric study over scattering ratio is presented for a representative problem with asymmetric, isotropic boundary conditions. Future efforts will focus on further solution verification, extension to 2D and 3D geometries, and comparisons to alternative bidirectional formulations.

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