A New Analytical Model to Evaluate Dancoff Factors in Stochastic Media

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INTRODUCTION

Evaluation of Dancoff factors for fuel particles in gas cooled reactors has been a research topic for half a century [1-5]. The accurate prediction of Dancoff factors plays important roles in the reactor analysis, especially for calculating the resonance integral in a fuel kernel accounting for the effect of the stochastic distribution of surrounding fuel particles. This becomes even more important for the analysis of current light water reactors loaded with TRISO fuels [6,7]. Many analytical formulas have been derived for calculating average Dancoff factors of fuel particles that are randomly distributed in infinite and finite media.

Recently, analytical expressions for Dancoff factors in stochastic media were obtained based on chord method, and were applied to analyze stochastic distribution of fuel particles in Very High Temperature Gas-cooled Reactors (VHTR's) [5]. A single-sphere model was proposed to calculate infinite medium, intra-finite medium and inter-finite medium Dancoff factors. However, this model ignores coating regions in fuel particles and assumes that fuel kernels, instead of fuel particles, are randomly distributed in fuel pebbles or fuel compacts. This implies that fuel kernels can have direct contact with each other, which is not a physically realistic configuration, even though the Dancoff factors were predicted with acceptable accuracy based on this model.

A new and more realistic dual-sphere model is developed in this paper for the evaluation of Dancoff factors in order to account for the coating regions. This new model is based on the assumption of an exponential chord length probability density function (PDF) between fuel kernels, similar to the previously developed single-sphere model [5]. However, in the new model, the existence of the coating region outside the fuel kernels is accounted for.

In this paper, we will show derivations of the new dual-sphere model and its applications to evaluating infinite medium, intra-pebble and intra-compact Dancoff factors of fuel particles.

METHODOLOGY DESCRIPTION

Analytical expressions have been obtained for Dancoff factors of fuel particles in infinite and finite media based on the chord method [5]. These expressions are listed below:

$$C^{\infty} = \int f(l) \exp(-l/\lambda) dl, \tag{1}$$

and

$$C^{intra} = \frac{1}{<L>} \int dL F(L) \int_0^L dl \int_0^l dl' f(l') \exp(-l'/\lambda). \tag{2}$$

where λ is the mean free path in background matrix material. In addition, $f(l)$ is the PDF of chord length between fuel kernels, and $F(L)$ is the PDF of chord length within a finite volume, such as a fuel pebble zone or a fuel compact region in VHTR's. $<L>$ is the mean chord length within a finite volume. As long as $f(l)$ can be accurately determined, the average Dancoff factors of fuel particles randomly distributed in infinite and finite media can be analytically evaluated. Previously, a single-sphere model was proposed for this evaluation [5]:

$$f^s(l) = (1/<l>) \exp(-l/<l>), \ 0 < l < \infty, \tag{3}$$

where superscript s implies a single-sphere model. $<l>$ is the mean chord length between fuel kernels and can be theoretically derived as[5]:

$$<l> = 4r/3 \cdot (1 - frac')/frac', \tag{4}$$

where r and $frac'$ are the radius and volume packing fraction of fuel kernels. Normally, a volume packing fraction (denoted as $frac$) of fuel particles, instead of fuel kernels, is provided as an important reactor design parameter. Given the radius of fuel particles R, $frac'$ can be calculated by: $frac'=frac \cdot (r/R)^3$. The single-sphere model in Eq. (3) allows fuel kernels to touch each other and ignores the coating regions. To improve upon this model, and to account for realistic configurations, a new dual-sphere model for $f(l)$ is developed. The new model still assumes an exponential function. However, the chord length has a minimum value t:

$$f^d(l) = \begin{cases} 0, & 0 < l < t, \\ \alpha \exp(-l/\beta), & t < l < \infty, \end{cases} \tag{5}$$

where superscript d represents a dual-sphere model and $t=2 \cdot (R-r)$. Parameters α and β can be determined by requiring Eq. (5) to satisfy the following two conditions.

$$\int_0^{\infty} f^d(l) dl = 1, \tag{6}$$

and

$$\int_0^{\infty} l f^d(l) dl = <l>, \tag{7}$$

where $<l>$ is determined by Eq. (4). After some straightforward calculations, Eq. (5) becomes:

$$f^d(l) = \begin{cases} 0, & 0 < l < t, \\ \dfrac{1}{<l>-t}\exp(-\dfrac{l-t}{<l>-t}), & t < l < \infty. \end{cases} \quad (8)$$

Eq. (8) can then be introduced to Eqs. (1) and (2) to evaluate Dancoff factors.

Next, we apply the new model to calculate Dancoff factors for fuel particles in VHTR designs. In VHTR, fuel particles are randomly distributed in either a fuel compact or a fuel pebble. Three types of Dancoff factors are evaluated and compared to the Monte Carlo reference simulations: average infinite medium, intra-pebble and intra-compact (infinite height cylinder) Dancoff factors as a function of fuel particle volume packing fraction.

For the intra-pebble case, $F(L)$ is expressed by [5]:

$$F(L) = \frac{L}{2R_p{}^2}, \ 0 < L < 2R_p, \quad (9)$$

where R_p is the radius of the fuel zone in a fuel pebble.

For the intra-compact case, $F(L)$ is expressed by [5]:

$$F(L) = \frac{16R_c{}^2}{\pi L^3}\int_0^{x_0}\frac{x^4 dx}{\sqrt{\left(L^2/4R_c{}^2\right)-x^2}\sqrt{1-x^2}}, \quad (10)$$

where $x_0 = \begin{cases} 1 & L > 2R_c \\ \dfrac{L}{2R_c} & L < 2R_c \end{cases}$, and R_c is the radius of the fuel compact.

Values of parameters in Dancoff factor evaluations are listed in Table I.

Table I. Values of system parameters [5]

r(cm)	R(cm)	$1/\lambda$(cm^{-1})	R_p(cm)	R_c(cm)
0.0175	0.0390	0.4137	2.5	0.6225

In Monte Carlo reference calculations, fuel particles are randomly packed in a large cubic container, a spherical fuel zone and a cylindrical fuel compact, respectively, based on a Random Sequential Addition (RSA) algorithm [8]. A total of 1,000 physical realizations are generated for each type of Dancoff factor calculations. In each realization, neutrons are emitted at surface of fuel kernels with cosine current angular distribution. They then are tracked until a collision occurs in the background or in the coating regions before entering another fuel kernel or before leaking out of the system. The probability that a neutron can reach another fuel kernel without any collision is calculated as the average fuel kernel Dancoff factor in the stochastic system. The system packing fraction ranges from 2% to 28% with an increment of 2%. Two extra packing fractions 5.76% and 28.92% are also studied, corresponding to pebble bed VHTR and prismatic VHTR designs, respectively [5].

By introducing expressions $f(l)$ and $F(L)$ into Eqs. (1) and (2), analytical solutions based on single-sphere model and dual-sphere model are both calculated and compared with Monte Carlo references. In addition to data plotting, the average relative errors over the studied packing fraction range will be calculated as a measure of accuracy.
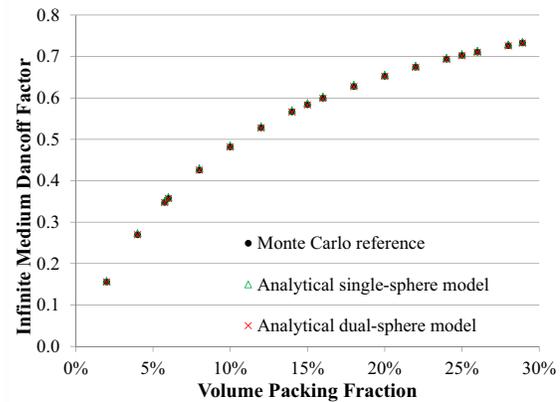
RESULTS AND ANALYSIS

Fig. 1. Average Dancoff factor of fuel kernel in infinite medium

In Monte Carlo simulations for infinite medium Dancoff factor, neutrons are emitted from fuel kernels located around center region of a large cubic container. The size of the container is adjusted so that neutrons have zero first-flight leakage probability, which mimics an infinite medium system. Fig. 1 shows the solutions evaluated at different packing fractions using Monte Carlo and analytical methods. Both analytical models predict very accurate solutions. The average relative errors are .70% and -.09% respectively for single-sphere and dual-sphere models.
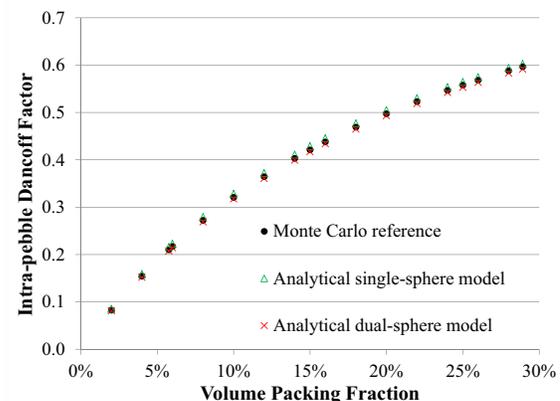

Fig. 2. Average Dancoff factor of fuel kernel in a fuel pebble

In Monte Carlo simulations for the intra-pebble Dancoff factor, neutrons are emitted from fuel kernels uniformly located inside the fuel zone of a fuel pebble. Fig. 2 shows the Dancoff factors evaluated at different packing fractions using Monte Carlo and analytical methods. Both analytical models predict accurate solutions. The average relative errors are 1.91% and -.101% respectively for single-sphere and dual-sphere models. At the packing fraction of 5.76%, the relative errors for the single-sphere model and the dual-sphere model are 2.73% and -1.32%, respectively.
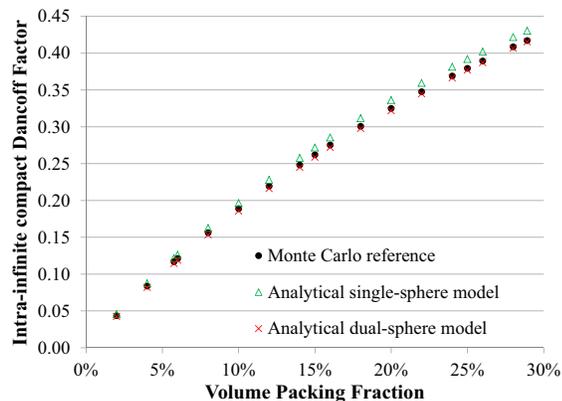

Fig. 3. Average Dancoff factor of fuel kernel in an infinite-height fuel compact

In Monte Carlo simulations for the intra-compact (infinite height cylinder) Dancoff factor, a fuel compact system with a height of 200 cm is employed. Neutrons are emitted from fuel kernels located around central region in the compact so that zero neutron first-flight leakage occurs at the top and bottom boundaries, which mimics an infinite height cylinder system. Fig. 3 shows the Dancoff factors evaluated at different packing fractions using Monte Carlo and analytical methods. It can be seen that dual-sphere model predicts more accurate solutions than the single-sphere model over the studied range of packing fraction. The average relative errors are 3.74% and -1.25%, respectively, for single-sphere and dual-sphere models. At the packing fraction of 28.92%, the relative errors are 3.17% and -0.42% for each model, respectively.

From Figs. 1-3, some common features can be observed. For all cases, the single-sphere model overestimates the Dancoff factor while the dual-sphere model underestimates it. In the single-sphere model, since the coating is not accounted for, there is an increase in the possibility that a neutron can reach fuel kernels without crossing a coating region. Therefore, the solutions predicted by this model become larger than the reference results. The dual-sphere model accounts for the coating region and predicts more accurate solutions than the single-sphere model. However, the consistent underestimation of the Dancoff factor by the dual-sphere model implies that the exponential chord length distribution assumption between fuel kernels overestimates the probability density for short chord lengths. Future work will focus on seeking a piecewise chord length PDF (a composite linear-exponential PDF [9]) to improve the accuracy of the dual-sphere model.

CONCLUSIONS

By assuming an exponential chord length PDF and imposing a key constraint that a minimum distance exists between fuel kernels in a stochastic system, a new dual-sphere model is developed for the evaluation of Dancoff factors for fuel particles in an infinite medium, a fuel pebble, and an infinite height fuel compact over a range of packing fractions. The new model has been verified to predict more accurate Dancoff factors then a previously developed single-sphere model. It provides an accurate and fast analytical method to calculate Dancoff factors, which are used on a routine basis in reactor analysis codes to analyze gas-cooled or light water reactors loaded with TRISO fuels.

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