A Novel Extension of Chord Length Sampling Method for TRISO-Type Applications

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INTRODUCTION

Chord length sampling (CLS) method has been shown to be an accurate and efficient Monte Carlo method for solving radiation transport problems in stochastic media [1-11]. Examples of stochastic media include reactor systems consisting of multi-type fuel particles that have different sizes. In the analysis of these reactor systems using the CLS method, fuel particles are sampled on-the-fly and only one fuel particle's position is stored during neutron's random walk. Moreover, only one simulation is needed, instead of many simulations over different physical realizations. Thus, the intensive computation time and large computer memory required by the conventional method of explicitly modeling the location of each fuel particle are greatly decreased.

In CLS simulations, a chord length probability density function (PDF) is used to sample the next fuel particle. Within fuel particles, the conventional Monte Carlo algorithm is applied [3-5]. A fuel particle, such as TRISO fuel used in the gas-cooled reactors or potentially in light water reactors, consists of a small fuel kernel region and four coating layers. The coating layers are optically thin, and can be homogenized into one coating region or totally smeared with the background without introducing neutronic analysis errors. This implies that explicitly tracking neutrons in coating layers or the homogenized coating region may not be necessary. Thus, we propose to directly sample fuel kernels, instead of fuel particles, on the fly in the CLS simulations. Within fuel kernels, regular Monte Carlo method is still used. In this way, the coating region becomes part of background from neutron's perspective. However, in realistic configurations, fuel kernels are distributed and separated by a minimum distance between each other due to coating regions. To keep the accuracy, this realistic physical model has to be captured in the CLS simulations, which can be achieved by developing a new chord length PDF that accounts for the minimum distance between fuel kernels.

In this paper, we develop a new chord length PDF model to directly sample fuel kernels in CLS simulations. This model shows that a higher efficiency can be obtained without sacrificing computational accuracy.

METHODOLOGY DESCRIPTION

An exponential chord length PDF has been used to sample the next fuel particle in previous CLS simulations:

$$f_p(l_p) = (1/ < l_p >) \exp(-l_p / < l_p >),\ 0 < l_p < \infty, \quad (1)$$

where subscript p means fuel particle. $<l_p>$ is the mean chord length between fuel particles and can be theoretically derived [4]:

$$< l_p > = 4R / 3 \cdot (1 - frac) / frac, \quad (2)$$

where R and $frac$ are the radius and volume packing fraction of fuel particles. When CLS is applied to sample fuel kernels, we still assume an exponential chord length PDF. However, the chord length has a minimum value d:

$$f_k(l_k) = \begin{cases} 0, & 0 < l_k < d, \\ \alpha \exp(-l_k / \beta), & d < l_k < \infty, \end{cases} \quad (3)$$

where d=2·(R-r) and r is the radius of fuel kernel. Parameters α and β can be determined by requiring Eq. (3) to satisfy the following two conditions.

$$\int_0^\infty f(l_k) dl_k = 1, \quad (4)$$

and

$$\int_0^\infty l_k f(l_k) dl_k = < l_k >, \quad (5)$$

where $<l_k>$ is the mean chord length between fuel kernels and can be calculated in a similar way as Eq. (2):

$$< l_k > = 4r / 3 \cdot (1 - frac') / frac', \quad (6)$$

where $frac'$ is the fuel kernel volume packing fraction and can be calculated by $frac'$=$frac$·$(r/R)^3$. After some straightforward calculations, Eq. (3) becomes:

$$f_k(l_k) = \begin{cases} 0, & 0 < l_k < d, \\ \dfrac{1}{< l_k > - d} \exp\left(-\dfrac{l_k - d}{< l_k > - d}\right), & d < l_k < \infty. \end{cases} \quad (7)$$

Eq. (7) will be the new chord length PDF that is used to sample fuel kernels in the CLS simulations.

Next, a series of one-group eigenvalue problems are simulated to test accuracy and efficiency of the new CLS model by comparing the predicted effective multiplication factor (k_{eff}) and the spatial distribution of fuel kernel flux (a measure of fission power) over the container with the reference solutions. The simulations are performed in a 10*10*10cm cubic container with vacuum boundary conditions. Fuel particles (R=0.039cm), consisting of a fuel kernel (r=0.0175cm) and a coating layer, are randomly packed inside the container at volume packing fractions ranging from 5% to 60%. The cross sections for the background and the coating layer materials are the same: $\Sigma_{t,1}$=0.4137cm^{-1} and $\Sigma_{s,1}$=0.4137cm^{-1}. Fuel kernels have the fissile material with the cross sections at $\Sigma_{t,2}$=400cm^{-1}, $\Sigma_{f,2}$=80cm^{-1}, $\Sigma_{s,2}$=200cm^{-1}, and ν=2.5.

In reference simulations, the Random Sequential Addition (RSA) method is used to pack fuel particles in the container at the packing fractions from 5% to 30%. A dynamic-based packing method [12] is employed to pack particles at the packing fractions from 35% to 60%. Fig. 1 shows a physical realization of fuel particles packed in the container. A total of 100 realizations are generated for k_{eff} prediction, and 1,000 realizations are generated for calculating fuel kernel flux distribution (only at the packing fraction of 30%). Conventional Monte Carlo method is employed for each realization and the solutions are ensemble-averaged over realizations.

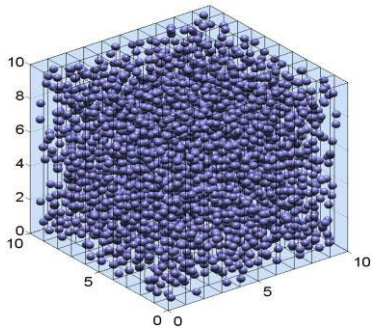


Fig. 1. Fuel particles packed in a cubic container.

In CLS simulations, two models based on Eq. (1) and Eq. (7) are applied and the solutions are compared with reference solutions in terms of accuracy and efficiency.

RESULTS AND ANALYSIS

A total of 1 million neutrons per cycle and 50 inactive and 250 active cycles are used in both reference and CLS simulations for the calculation of k_{eff}, producing a standard deviation within 1e-4 over the studied packing fractions. Table I compares the accuracy of the new CLS model with the reference. Table II compares the time efficiency of two CLS models based on Eqs. (1) and (7). All the calculations are performed on a desktop with Intel Xeon X3430 2.40GHz processor in a single thread mode.

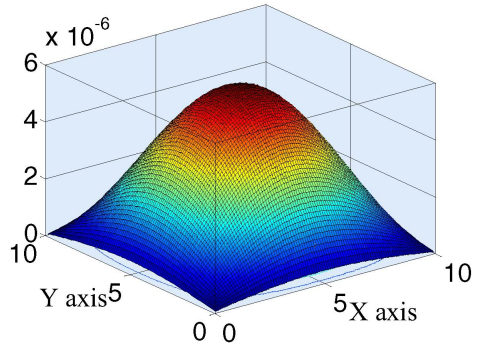
Table I. Accuracy comparisons in predicting k_{eff}

frac	Reference	CLS Eq. (1)	CLS Eq. (7)	Rel. error
5%	0.9473	0.9468	0.9461	-0.13%
10%	0.9696	0.9694	0.9688	-0.08%
15%	0.9807	0.9805	0.9800	-0.07%
20%	0.9862	0.9865	0.9862	0.00%
25%	0.9900	0.9902	0.9899	-0.01%
30%	0.9925	0.9924	0.9922	-0.03%
35%	0.9939	0.9938	0.9937	-0.02%
40%	0.9949	0.9949	0.9948	-0.01%
45%	0.9957	0.9957	0.9956	-0.01%
50%	0.9963	0.9962	0.9962	-0.01%
55%	0.9967	0.9966	0.9967	0.00%
60%	0.9970	0.9970	0.9970	0.00%

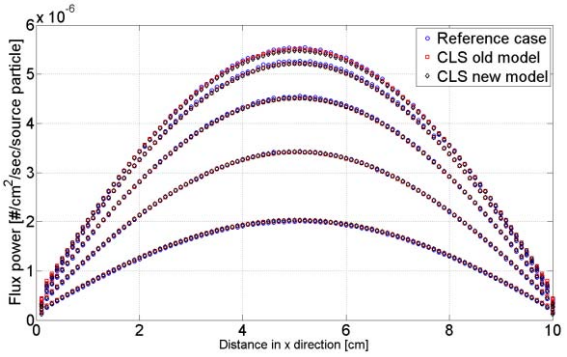
Table II. Efficiency comparisons in predicting k_{eff}

frac	CLS Eq.(1) in minute	CLS Eq. (7) in minute	Speedup
5%	4.74	3.54	1.34
10%	5.04	3.54	1.42
15%	5.34	3.72	1.44
20%	5.76	3.72	1.55
25%	5.58	3.54	1.58
30%	5.82	3.66	1.59
35%	6.00	3.78	1.59
40%	5.94	3.90	1.53
45%	6.00	3.78	1.59
50%	5.94	3.78	1.57
55%	6.00	3.78	1.59
60%	6.12	3.78	1.62

From Table I, the relative errors between the new CLS model and the reference are less than 0.15% over all the configurations. Compared to the old CLS model, the new model shows the same accuracy at higher packing fractions (>15%). At lower packing fractions (<15%), the old model has more accurate predictions than the new model but both predictions are within acceptable accuracy. Table II shows a consistent speedup of around 1.5 for the new CLS model compared to the old one over all the packing fractions.



(a) Fuel kernel flux distribution in reference simulation



(b) Flux distribution comparisons over cells located at Y=1 (lowest curves), 2, 3, 4, and 5 (highest curves) cm

Fig. 2. Fuel kernel flux distribution comparison.

Flux spatial profile is another important criterion to evaluate the accuracy of CLS simulations. To calculate flux distribution, the container is divided into 100*100*1 cells as shown in Fig. 1. Collision estimator is used to tally the volume average flux of fuel kernels in each cell. A total of 10M neutrons per cycle with 1,000 inactive and 4,000 active cycles are used to guarantee the fission source convergence and lead to a relative standard deviation of 1e-3 in each cell. Fig. 2 (a) shows the flux distribution in the reference simulation, which is an ensemble average over 1,000 realizations. Due to the symmetry, we compare flux distributions over cells only at five Y axis locations, which are shown in Fig. 2 (b). The average absolute relative errors of these flux curves are listed in Table III. It shows that both CLS models predict very accurate flux distribution compared to the reference, with the absolute relative errors within 0.5% for inner regions. We also calculate the average absolute relative error over the whole container (10,000 cells): 0.99% for the old model, and 1.20% for the new model. The relative errors of the total flux over the system are only 0.012% for the old model, and -0.008% for the new model. The new CLS model is demonstrated to be an accurate model in predicting both the k_{eff} value and the flux distribution in a random media system consisting of fuel particles.

Table III. Average relative errors for both CLS models

Location of cells	Average relative errors between CLS and reference simulations over cells	
	old model in CLS	new model in CLS
Y=1 cm	1.19%	1.46%
Y=2 cm	0.12%	0.35%
Y=3 cm	0.29%	0.17%
Y=4 cm	0.46%	0.41%
Y=5 cm	0.51%	0.49%

Similar to k_{eff} calculations, the computational time for flux predictions is significantly reduced using the new CLS model, with a total of 10.7 hours versus 16.1 hours using the old CLS model. The speedup is as high as 1.5.

CONCLUSIONS

A new chord length distribution model for CLS simulation is proposed in this paper. By sampling the chord length between fuel kernels instead of fuel particles, the CLS method using the new model shows a significant increase in computational efficiency, without sacrificing accuracy. We apply the CLS methods to solving eigenvalue problems in a 3-D stochastic medium system, and observe very good accuracy in predicting the multiplication factor and flux power distribution. It demonstrates that the new model for CLS simulation with derived theoretical chord length PDF in this paper can be more efficient for practical applications, such as the analysis of the stochastic distribution of fuel particles in gas-cooled reactors or in potential light water reactors with inherently safety features. This is especially true for the full core analysis.

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