Application of Chord Length Sampling to Analyzing Reactor Systems with Multi-type Multi-size Fuel Particles

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INTRODUCTION

Previous work about chord length sampling (CLS) method focused on the analysis of reactor systems with single-type mono-sized [1-3] or poly-sized [4, 5] fuel particles. CLS has been shown an accurate and efficient method to analyze such reactor systems.

In some advanced reactor designs, multi-type fuel particles are used and they are randomly mixed together in the fuel region. Typical example is the Deep Burn concept in Gas Turbine Modular Helium Reactor (GT-MHR) using two types of TRISO particles called transmutation fuel and driver fuel [6] . Also, there has been great interest in adopting thorium fuel as a supplement to limited uranium resource since 1960s. Thorium fuel particle and uranium fuel particle are both employed in reactor systems. Typical example is the Fort St. Vrain (FSV) reactor [7] . The mixture of two or more types of fuel particles in a random configuration presents more challenges to current reactor analysis capability, especially for the global analysis.

In this paper, we apply CLS method to analyze reactor systems with multi-type multi-size fuel particles. The capability of CLS method for the analysis of stochastic distribution of two types of fuel particles in a cubic system is demonstrated.

METHODOLOGY DESCRIPTION

In the analysis of stochastic system with single-type fuel particles, CLS is used to sample the fuel particle "on the fly" based on a chord length probability distribution function (PDF) [8]. When one comes to multi-type fuel particle systems, due to the difference in fuel particle size for each fuel type, a new chord length PDF should be formulated. Also, additional sampling is needed to determine the type and the size of each fuel particle when CLS is performed.

In current research, for simplicity, we study a system consisting of two types of fuel particles with fixed radii R_1 and R_2. One represents a fissile fuel particle and the other represents a fertile fuel particle. Given the volume packing fraction for each fuel type, $frac_1$ and $frac_2$, the size distribution of all the fuel particles in the system can be written as

$$s(r) = p_1\delta(r - R_1) + p_2\delta(r - R_2), \tag{1}$$

where $p_1+p_2=1$ and $p_1/p_2=(frac_1*R_2^3)/(frac_2*R_1^3)$. p_i (i=1 or 2) represents the probability of finding the fuel type i in the system. When $R_1=R_2$, $p_1=frac_1/frac$ and $p_2= frac_2/frac$.

In CLS, a chord length PDF in the background material is derived by assuming an exponential function

$$f(l_b) = (1/<l_b>)e^{-l_b/<l_b>}, \tag{2}$$

where $<l_b>$ is the mean chord length in the background material. The validation of the above assumption has been demonstrated by many researchers in mono-sized and poly-sized sphere systems [9]. Generally, $<l_b>$ can be theoretically calculated by an infinite medium scaling relationship:

$$<l_b> = <l_s>(1 - frac)/frac, \tag{3}$$

where $<l_s>$ is the mean chord length in fuel particles and $frac=frac_1+frac_2$. When $R_1=R_2=r$, $<l_s>=4r/3$. When R_1 is not equal to R_2, $<l_s>$ must be calculated by deriving the PDF of l_s. Following the derivation procedure for poly-sized sphere system [9], $<l_s>$ can be calculated as

$$<l_s> = (4/3)(p_1R_1^3 + p_2R_2^3)/(p_1R_1^2 + p_2R_2^2). \tag{4}$$

By substituting the parameter back to Eqs. (2) and (3), the chord length PDF used to sample the next fuel particle can be obtained. The probability that the sampled fuel particle is type i can be calculated based on [9]:

$$p_i^* = p_iR_i^2 \Big/ \left(p_1R_1^2 + p_2R_2^2\right), \tag{5}$$

where i=1 or 2. When $R_1=R_2$, $p_i^*=p_i$.

Next, a series of eigenvalue problems are simulated at different cross sections and total volume packing fractions. A list of parameter values are shown in Table I. The fissile fuel particle has a fixed radius of R_1=0.0375cm in all the simulation scenarios. Cases 1 to 3 represent three sets of cross sections that have the values of 0.1, 1.0 and 10 for the ratio of mean chord length and mean free path ($<l_{fk}>/\lambda_{fk}$) in the fuel particle, corresponding to the scenarios that neutrons have rare, a few, and many interactions within the fissile particle. The fertile fuel particle has a varying radius R_2 from $0.5R_1$, R_1 to $2R_1$, denoted as cases a to c, and has fixed cross sections at $\Sigma_{t,fk2}$=20cm^{-1}, $\Sigma_{a,fk2}$=2cm^{-1}, $\Sigma_{s,fk2}$=18cm^{-1}. A total of combined 9 cases are studied at the total volume packing fraction of 6% and 30%. Case 3c is also simulated at the packing fraction of 60%. The volume packing fractions (equivalent to mass) for both fuel particle types are fixed at $frac_1$=0.75*$frac$ and $frac_2$=0.25*$frac$ for all the simulation cases.

In reference simulations, fuel particles are first packed inside an 8*8*8cm cube, and then regular Monte

Carlo (MC) method is used to calculate the eigenvalue problems with vacuum boundary conditions applied. The Random Sequential Addition (RSA) method is used to sample the particle (center) position in the container at the packing fractions of 6% and 30%. A dynamic-based packing method [10] is employed to pack particles at the 60% packing fraction. After a fuel particle is sampled, its type is determined based on the probability p_i in Eq. (1). Figure 1 shows one realization of two types of fuel particles packed in a 2*2*2cm container. In the eigenvalue problem, the effective multiplication factor (k_{eff}), and the volume-average flux ratio of fuel particles to the background material (defined as the particle shielding factor) are calculated. The results are ensemble-averaged over a total of 100 realizations.

In the CLS simulation, fuel particle is sampled on the fly based on Eq. (2) and the type of fuel particle is determined based on Eq. (5). When a neutron leaves a fuel particle, no fuel particle is explicitly modeled until the neutron enters next one. Previous research [11] has shown this is the correct procedure in CLS.

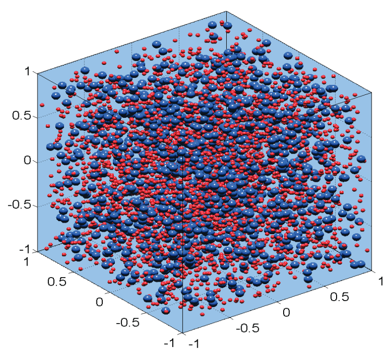

Figure 1. Two types of fuel particles packed in a 2*2*2cm cube at 6% total volume packing fraction

RESULTS AND ANALYSIS

A total of 1 million neutrons per cycle with 50 inactive and 250 active cycles are used for both reference and CLS simulations. Table II compares the results of k_{eff}'s and fuel particle shielding factors. The absolute relative errors are less than 0.5%, 2.5% and 4.5% for k_{eff}'s, fissile and fertile shielding factors, respectively. Larger errors occur at case c (larger size of the fertile particle) due to the boundary effect [5]. It is observed that in the CLS simulation for these cases, there is an appreciable reduction in the packing fraction of fertile particles so causing the reduction in the shielding factors. It is expected that if the packing fraction correction is applied, more accurate solutions can be obtained. This is being investigated and will be reported in the future.

It is interesting to see that similar k_{eff}'s are obtained at the same volume packing fractions in cases 1-3. This shows that k_{eff}'s are not sensitive to the size of fertile fuel particles, when the total mass (volume packing fraction) of the fissile fuel particle is the same.

In the computation of high packing fraction (60%) case, case 3c is studied. This configuration is a typical representative of FSV design. It is found the calculated k_{eff} and shielding factors are very close between CLS and reference cases.

CONCLUSIONS

By applying the CLS method to solving eigenvalue problems in a 3D stochastic medium packed with two types of fuel particles, very good accuracy has been observed in predicting the multiplication factor and fuel particle shielding factor in each fuel particle type compared to the reference solutions. It demonstrates that the CLS method with derived theoretical chord length PDF's in this paper can be applied to analyze reactor systems with multi-type multi-size fuel particles, such as Fort St. Vrain reactor.

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Table I. Material and geometry parameters in reference and CLS simulations

Fissile	$\Sigma_{t,\mathrm{fk1}}$	$\Sigma_{a,\mathrm{fk1}}$	$\Sigma_{f,\mathrm{fk1}}$	$\nu\Sigma_{f,\mathrm{fk1}}$	$\Sigma_{s,\mathrm{fk1}}$	$\langle l_{\mathrm{fk1}}\rangle/\lambda_{\mathrm{fk1}}$	**Fertile**	R_2	$\langle l_{\mathrm{fk2}}\rangle/\lambda_{\mathrm{fk2}}$
1	8.0	4.0	1.6	3.24*1.6	4.0	0.1	**a**	$0.5R_1$	2
2	80	40	16	3.24*16	40	1.0	**b**	R_1	1
3	800	400	160	3.24*160	400	10	**c**	$2R_1$	0.5

Table II. Effective multiplication factors and fuel particle shielding factors (1σ = 1e-4)

Packing fraction		6%			30%		
Case	Method	k_{eff}	Fissile(L) & fertile(R) shielding factor		k_{eff}	Fissile(L) & fertile(R) shielding factor	
1a	Reference	0.5172	0.0533	0.0156	0.9063	0.3318	0.1037
	CLS	0.5130	0.0523	0.0155	0.9039	0.3288	0.1042
	Rel. Error	-0.22%	-1.88%	-0.64%	-0.27%	-0.90%	0.48%
1b	Reference	0.5116	0.0534	0.0151	0.9073	0.3317	0.1000
	CLS	0.5083	0.0525	0.0149	0.9056	0.3293	0.1004
	Rel. Error	-0.17%	-1.69%	-1.33%	-0.15%	-0.72%	0.40%
1c	Reference	0.5056	0.0535	0.0142	0.9066	0.3312	0.0962
	CLS	0.5033	0.0526	0.0136	0.9122	0.3294	0.0919
	Rel. Error	-0.12%	-1.68%	-4.23%	0.51%	-0.54%	-4.47%
2a	Reference	1.1703	0.0533	0.0156	1.2506	0.3340	0.1046
	CLS	1.1676	0.0522	0.0155	1.2501	0.3304	0.1042
	Rel. Error	-0.32%	-2.06%	-0.64%	-0.04%	-1.08%	-0.38%
2b	Reference	1.1694	0.0533	0.0153	1.2521	0.3343	0.0997
	CLS	1.1673	0.0523	0.0150	1.2514	0.3308	0.1007
	Rel. Error	-0.25%	-1.88%	-1.96%	-0.09%	-1.05%	1.00%
2c	Reference	1.1691	0.0534	0.0143	1.2536	0.3347	0.0954
	CLS	1.1675	0.0523	0.0137	1.2542	0.3304	0.0928
	Rel. Error	-0.19%	-2.06%	-4.20%	0.05%	-1.29%	-2.73%
3a	Reference	1.2853	0.0659	0.0158	1.2912	0.3650	0.1022
	CLS	1.2851	0.0650	0.0156	1.2913	0.3597	0.1007
	Rel. Error	-0.03%	-1.37%	-1.27%	0.01%	-1.45%	-1.47%
3b	Reference	1.2851	0.0664	0.0146	1.2913	0.3650	0.1021
	CLS	1.2849	0.0652	0.0143	1.2913	0.3597	0.1007
	Rel. Error	-0.03%	-1.81%	-2.06%	0.00%	-1.45%	-1.37%
3c	Reference	1.2851	0.0666	0.0146	1.2915	0.3650	0.0949
	CLS	1.2851	0.0649	0.0143	1.2915	0.3571	0.0929
	Rel. Error	0.00%	-2.55%	-2.06%	0.00%	-2.16%	-2.11%

frac=60% Case 3c	k_{eff} (1σ = 1e-4)	Fissile(L) and Fertile (R) Shielding Factor (1σ = 1e-5)	
Reference/CLS	1.2927/1.2922	1.23821/1.22906	0.26055/0.25369
Relative Error	-0.04%	-0.739%	-2.633%