Determinación de la temperatura y tiempos de equilibrio de electrones utilizando un modelo de nube de electrones con frecuencias de colisión y coeficientes de tasa calculados por BOLSIG+

Autores: Elise N. Pusateri\textsuperscript{1,2,*}, Heidi E. Morris\textsuperscript{2}, Eric M. Nelson\textsuperscript{2}, y Wei Ji\textsuperscript{1,*}

\textsuperscript{1}Departamento de Ingeniería Mecánica, Aeronáutica y Nuclear, Instituto de Tecnología de Rensselaer, Troy, Nueva York, EE. UU.

\textsuperscript{2}X-División, Laboratorio Nacional de Los Alamos, Los Alamos, Nuevo México, EE. UU.

*Correspondencia dirigir a: Elise N. Pusateri (elisep@lanl.gov) y Prof. Wei Ji, (jiw2@rpi.edu)

Puntos clave:
- Un modelo de nube de electrones se mejora utilizando BOLSIG+ y se actualizan las secciones cruzadas.
- El modelo mejorado se valida comparando los datos experimentales.
- La evolución de la temperatura se muestra sensible a las secciones cruzadas de escattering.
Abstract

Electromagnetic Pulse (EMP) events produce low-energy conduction electrons from Compton electron or photoelectron ionizations with air. It is important to understand how conduction electrons interact with air in order to accurately predict EMP evolution and propagation. An electron swarm model can be used to monitor the time evolution of conduction electrons in an environment characterized by electric field and pressure. Here, a swarm model is developed that is based on the coupled ordinary differential equations (ODEs) described by Higgins, Longmire and O’Dell (1973), hereinafter HLO. The ODEs characterize the swarm electric field, electron temperature, electron number density, and drift velocity. Important swarm parameters, the momentum transfer collision frequency, energy transfer collision frequency, and ionization rate, are calculated and compared to the previously reported fitted functions given in HLO. These swarm parameters are found using BOLSIG+, a two term Boltzmann solver developed by Hagelaar and Pitchford (2005), which utilizes updated cross sections from the LXcat website created by Pancheshnyi et al. (2012). We validate the swarm model by comparing to experimental effective ionization coefficient data in Dutton (1975) and drift velocity data in Ruiz-Vargas et al. (2010). In addition, we report on electron equilibrium temperatures and times for a uniform electric field of 1 StatV/cm for atmospheric heights from 0 to 40 km. It is shown that the equilibrium temperature and time are sensitive to the modifications in the collision frequencies and ionization rate based on the updated electron interaction cross sections.

Keywords:
Swarm model, electron collision frequency, electron energy distribution, electron temperature, electron cross sections, equilibrium

1. INTRODUCTION

Propagation of an EMP through the atmosphere depends on the production and evolution of low-energy conduction electrons. Compton electrons and photoelectrons produce conduction electrons when they slow in air. Conduction electrons also multiply via cascade ionization. These electrons can typically have energies on the order of a few eV. A sufficient number of conduction electrons will damp or alter the EMP through conduction current. Therefore, the accurate prediction of the conduction electron transport in the electric field generated by the
EMP would offer a better understanding of the EMP propagation phenomenon. One of the key efforts is to develop conduction current calculation models with high fidelity.

Many EMP simulation codes (see [Erkkila, 1967; Longley and Longmire, 1969; Radasky and Knight, 1971; Dalich and Granzow, 1974; Longmire, 1978; Roussel-Dupre, 2005; Eng, 2011]) use an equilibrium ohmic model for computing the conduction current \( J_s = \sigma(E)E \), where \( \sigma(E) \) is the electron conductivity as a function of the local, instantaneous EMP electric field \( E \). \( \sigma(E) \) is calculated through the equilibrium cascade ionization rate \( C_i(E) \) and electron mobility \( \mu(E) \). The conduction electrons are assumed to be in equilibrium with \( E \), instantaneously reaching steady state without a transient process.

The equilibrium model works well when the equilibration time is short compared to the rise time or duration of the EMP. However, at high altitude, the conduction electron equilibration time can be comparable to or longer than the rise time or duration of the EMP [Roussel-Dupre, 2005]. At high altitude, the number density of the surrounding air decreases, and, therefore, the collision frequency decreases, so the electrons will take a longer time to reach equilibrium relative to lower altitudes. This matters, for example, when calculating the EMP propagating upward toward a satellite from a moderately high-altitude burst. In these scenarios, the equilibrium ionization rate becomes very large for even a modest electric field. The equilibrium model produces an unphysically large number of conduction electrons that prematurely and abruptly short the EMP in the simulation code. In reality, the conduction electrons do not have time to reach equilibrium during the EMP pulse. The ionization rate is much lower than the equilibrium model assumes, and the EMP pulse is not abruptly shorted. To overcome the limitations exhibited by the equilibrium ohmic model, various models that account for the equilibrium process of conduction electrons have been proposed and employed for EMP applications [e.g., Higgins et al., 1973; Carlile, 1979; Mayhall et al., 1991; Solovyev et al., 1999; Sprangle et al., 2004; Ray et al., 2006]. The most widely used one among these is the swarm model. This is primarily due to its low computational cost.

A swarm model is a fluid model that assigns a mean temperature to electrons and monitors their time evolution through use of various collision frequencies and reaction rates in conservation equations. The swarm model presented here evolves the thermal energy and drift velocity of the conduction electrons. The accuracy of these calculated transport coefficients
depends on the accuracy of the swarm parameters, the momentum and energy transfer collision frequencies and ionization rate, used in the swarm model. The determination of the swarm parameters through experimental or computational means is a major effort from past to current [e.g., Phelps, 1960; Kroll and Watson, 1972; Warman et al., 1984; Ali, 1988; Chen and Davidson, 2002] that warrants the application of swarm models.

In this paper we focus on developing a swarm model that uses accurate swarm model parameters to calculate transport coefficients, including the electric field, electron number density, electron drift velocity, and electron temperature. We use BOLSIG+ [Hagelaar and Pitchford, 2005] to evaluate swarm parameters using the latest electron interaction cross section data with air, and employ the published set of swarm equations presented in HLO which are shown in Section 2.1. The new momentum and energy transfer collision frequencies and ionization rate calculations are described in detail in Section 2.1.2. The swarm parameters published in HLO, which are based off of older data, are presented in Section 2.1.1 and are also employed in the swarm model. We then evaluate the equilibration time and equilibrium temperature using the new swarm parameters at various altitudes and compare these times and temperatures against those calculated using the rates published in HLO. The swarm model is validated with experimental effective ionization coefficient and drift velocity data in air from [Dutton, 1975] and [Ruiz-Vargas et al., 2010], respectively, in Section 3.4. Swarm model validation demonstrates that the swarm model can adequately calculate transport parameters while accounting for the time conduction electrons need to reach equilibrium with the electric field.

2. METHODOLOGY DESCRIPTION

2.1 The Electron Swarm Model

The swarm model HLO presents comprises four coupled, ordinary differential equations that monitor the time evolution of the low energy conduction electron number density $N_e(t)$, temperature $U_e(t)$, drift velocity $v_d(t)$, and electric field $E(t)$. Equation (1) describes the change in electron number density $N_e(t)$ at time $t$.

$$\frac{dN_e(t)}{dt} = S_i(t) + C_i(U_e)N_e(t) - \alpha(U_e)N_e(t),$$

(1)
where \( S_1(t) \) is the production rate of the low energy electrons due to the ionization process experienced by the high energy Compton electrons or photoelectrons. For this study, we track an arbitrary initial number density of conduction electrons, so \( S_1(t) = 0 \). These low energy conduction electrons can multiply by avalanche ionization with ionization rate coefficient \( C_1(U_e) \). The rate coefficient is usually determined by experiment or computation and then fitted or tabulated as a function of the electron temperature \( U_e \). Electrons can be removed from the swarm through 2-body attachment (\( e + O_2 \rightarrow O^- + O \)) and 3-body attachment (\( e + O_2 + M \rightarrow O_2^- + M \), where \( M \) is an additional molecule, \( N_2 \) or \( O_2 \)) to oxygen [Zhao et al., 1995]. \( \alpha(U_e) \) is the attachment rate coefficient, which is not considered in the HLO ODEs, and is calculated in this analysis by BOLSIG+ as a function of \( U_e \).

The energy conservation equation that describes the time rate of change of the electron temperature, \( U_e(t) \), is

\[
\frac{dU_e(t)}{dt} = \frac{2}{3} \gamma \nu_d(t)E(t) - \nu_w(U_e) \cdot [U_e(t) - U_{e,0}] + S_2(t) \frac{U_e(t) dN_e(t)}{N_e(t)} dt.
\]  

The first term on the right hand side of Equation (2) is the gain in energy from the electric field as electrons are accelerated, where the \( 2/3 \) factor converts kinetic energy to characteristic temperature. It should be noted that, since the centimeter–gram–second system (cgs) system is being used, we have \( \gamma = 1.6 \times 10^{-12} \text{ eV/erg} \). The second term is a loss in electron energy due to energy transfer collisions with the surrounding gas. The energy transfer is quantified by the energy transfer collision frequency \( \nu_w(U_e) \). In addition, \( S_2 = \frac{2}{3} \left( \frac{\bar{w}}{N_e(t)} \right) S_1 - w_c C_1(U_e) \) is the net energy transfer rate to the swarm through ionization by primary electrons less the energy loss when secondaries produce further ionization. The ionization potential of air, \( w_c \), is assumed to be 14 eV. It has been shown that, initially, conduction electrons have average energies, \( \bar{w} \), of approximately 8 eV [Longmire and Longley, 1973]. Therefore, the conduction electrons will start at a temperature \( U_e = (2/3) \times 8 \text{ eV} \approx 5 \text{ eV} \). We see again the \( 2/3 \) factor in \( S_2 \) converting kinetic energy to characteristic temperature. The term including \( dN_e / dt \) here and in Equation (3) arises from going from a single particle equation to an equation of electron number density.
The third equation is for electron drift velocity, \( v_d(t) \), comes about from Newton’s law that relates the rate of change in momentum to the applied force.

\[
\frac{dv_d(t)}{dt} = \frac{e}{m_e} E(t) - v_m(U_c) v_d(t) - \frac{v_s(t)}{N_e(t)} \frac{dN_e(t)}{dt}.
\]

(3)

There is a gain in drift velocity due to the electric field, \( E(t) \), (first term) and a loss due to collisions with the surrounding gas (second term). The momentum transfer collision frequency \( v_m(U_c) \) quantifies the collisional loss.

The change in the electric field, defined in the fourth equation below, is

\[
\frac{dE(t)}{dt} = -4\pi e J_s(t),
\]

(4)

where \( J_s(t) = (e/c) N_e(t) v_d(t) \) is the conduction current. Neglecting electron inertia, the electric field decays on a time scale inversely proportional to the conductivity \( \sigma(t) = eN_e(t) \mu(t) = eN_e(t) v_d(t) / E(t) \) of the medium. The parameter definitions are summarized in Table 1 below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e )</td>
<td>Electric charge, statC.</td>
<td>( S_2 )</td>
<td>Rate that energy is added to the swarm, eV/s.</td>
</tr>
<tr>
<td></td>
<td>( = 4.803 \times 10^{-10} ) statC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( m_e )</td>
<td>Electron mass, g.</td>
<td>( \bar{w} )</td>
<td>Average energy of ionization electrons, eV ( \approx ) 8eV</td>
</tr>
<tr>
<td></td>
<td>( = 9.11 \times 10^{-28} ) g</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| \( N_e \) | Number density of conduction electrons per unit volume, 
#/#cm\(^3\). | \( w_e \) | Ionization potential of air, eV \( \approx \) 14 eV |
| \( S_1 \) | Rate at which conduction electrons are produced, 
#/#cm\(^3\)·s. | \( \nu_w \) | Energy transfer collision frequency, s\(^{-1}\). |
| \( C_1 \) | Ionization rate, s\(^{-1}\). | \( \nu_m \) | Momentum transfer collision frequency, s\(^{-1}\). |
The time dependence of $N_e(t)$, $U_e(t)$, $v_d(t)$, and $E(t)$ can now be calculated by updating the swarm parameters $C_1(U_e)$, $\nu_w(U_e)$, and $\nu_m(U_e)$ at each time step, therefore giving $\sigma(t)$. This gives us the ability to accurately evaluate $\sigma$ for the physical scenario where the electrons do not have time to reach equilibrium, i.e. the conductivity and the conduction current are calculated based on the non-equilibrium electron distribution. This is an improvement over the equilibrium ohmic model, which calculates the secondary current dependent only upon the equilibrium distribution of electrons.

We solve the swarm model ODEs using a variant of a 4th order Runge-Kutta method called the Rosenbrock method. Kaps and Rentrop produced the first practical implementation of these methods, so these methods are also called Kaps-Rentrop methods [Press et al., 1992]. These methods are used to solve stiff sets of equations, which refers to the case where the scale of the independent variable changes rapidly. The method employs an adaptive time step algorithm with an error estimate computed from the difference between test solutions of high and low order.

### 2.1.1 Swarm Parameters

In HLO, $\nu_w$, $\nu_m$, and $C_1$ are given as fitted functions of data from the 1960s and are valid up to temperatures of 67 eV [Higgins et al., 1973]. The data for the collision frequencies is based directly on experimental measurements for $U_e < 3$ eV and calculated from cross section data for $U_e > 3$ eV. The data for the ionization rate is based on experimental data for $U_e < 3$ eV.
and cross section data for $U_e > 20$ eV. There was no data available for $C_i$ for $3$ eV < $U_e < 20$ eV, so the fit may be inaccurate in this temperature range. For $\nu_w$, $\nu_m$, and $C_i$ there was no data found beyond 67 eV in the HLO analysis. The collision frequency for a specific scattering reaction is determined by an evaluation of the power input by the electrons to the scattering process [Engelhardt and Phelps, 1963]. The energy transfer collision frequency is given by HLO as

$$\frac{\nu_e(U_e)}{N} = 1.0 \times 10^{-11} + \frac{2.43 \times 10^{-8} U_e^{3.22}}{(1 + 2.59 \times 10^7 U_e^{0.15})^{0.7}} + \frac{4.13 \times 10^{-10} U_e^{5.22}}{(1 + 0.072 U_e^{4.74})^{0.965}},$$

(5)

where $N$ is the number density of air molecules in /cm$^3$.

The momentum transfer collision frequency is fitted by HLO as

$$\frac{\nu_m(U_e)}{N} = \frac{1.25 \times 10^{-7} U_e^{0.935}}{(1 + 1.26 U_e^{1.67})^{3.75}},$$

(6)

The ionization rate is given by HLO as

$$\frac{C_i(U_e)}{N} = \frac{3.3 \times 10^{-14} U_e^{8.7}}{(1 + 5.87 \times 10^{-3} U_e^{5.5}) (1 + 1.29 \times 10^{-3} U_e^{5.2})} + 3.26 \times 10^{-7} e^{-44.08/U_e},$$

(7)

The method for the calculation of the updated swarm model parameters is presented in the next section.

2.1.2 Updated Swarm Model Parameters

Recently, efforts have been made to update the collision cross sections that are used in the modeling and simulation of electron transport in gases subject to an electric field. The collision frequencies and reaction rates used by the swarm model can be calculated by employing these newly evaluated elastic and inelastic cross sections up to electron energies of 10,000 eV [Phelps, 2014]. This is made possible by the collaborative, community based efforts from the Plasma Data Exchange Project, which created a repository, the LXcat website, where cross section data can be shared [Pancheshnyi et al., 2012]. On the LXcat website, cross section data is constantly evolving as contributors add and update information. BOLSIG+ solves the Boltzmann transport equation for the EEDF and uses this EEDF, along with the updated cross sections from the LXcat website, to calculate the collision frequencies and ionization rate [Hagelaar and Pitchford, 2005].
The general expression for the ionization rate is [Hagelaar and Pitchford, 2005]

\[
\frac{C_i(U_e)}{N} = \left(\frac{2\gamma}{m_e}\right)^{1/2} \int_0^\infty \varepsilon f(\varepsilon)Q_i(\varepsilon) d\varepsilon,
\]

where \( \varepsilon \) is the electron energy in eV, \( Q_i(\varepsilon) \) is the energy-dependent ionization cross section in \( \text{cm}^2 \), and \( f(\varepsilon) \) is the EEDF in \( \text{eV}^{-3/2} \). The EEDF is normalized,

\[
\int_0^\infty \varepsilon^{1/2} f(\varepsilon) d\varepsilon = 1.
\]

The 2- and 3-body attachment rate to oxygen is calculated in the same way as the ionization rate except using the attachment cross section rather than the ionization cross section \( Q_i(\varepsilon) \).

The \( U_e \) dependence in the ionization rate comes from relating the EEDF to the electron temperature \( U_e \) calculated by \( U_e = D / \mu \gamma \) [Frost and Phelps, 1962], where \( D \) is the diffusion coefficient and \( \mu \) is the electron mobility. The diffusion coefficient and the mobility are calculated through \( f(\varepsilon) \) by:

\[
\frac{D}{N} = \frac{1}{3} \left(\frac{2\gamma}{m_e}\right)^{1/2} \int_0^\infty \frac{\varepsilon}{Q_m(\varepsilon)} f(\varepsilon) d\varepsilon,
\]

\[
\frac{\mu}{N} = -\frac{1}{3} \left(\frac{2}{m_e k}\right)^{1/2} \int_0^\infty \frac{\varepsilon}{Q_m(\varepsilon)} \frac{\partial f}{\partial \varepsilon} d\varepsilon.
\]

The general expression for the momentum transfer collision frequency is [Engelhardt and Phelps, 1963]

\[
\nu_m(U_e) = \left[\left(\frac{2m_e}{9k}\right)^{1/2} \int_0^\infty \frac{\varepsilon}{Q_m(\varepsilon)} \frac{df(\varepsilon)}{d\varepsilon} d\varepsilon\right]^{-1},
\]

where \( Q_m(\varepsilon) \) is the effective momentum transfer cross section in \( \text{cm}^2 \). \( Q_m(\varepsilon) \) includes the elastic momentum transfer cross section and the sum of the inelastic cross sections.

The general expression for the energy transfer collision frequency is [Engelhardt and Phelps, 1963]
where $Q_j$ and $Q_{j'}$ are $j^{th}$ level inelastic excitation and de-excitation cross sections, respectively. $\varepsilon_j$ is the energy loss in eV associated with the $j^{th}$ level and $M$ is the molecular mass in g.

In this analysis, we assume the surrounding atmospheric air is composed of 1% Ar, 21% O$_2$, and 78% N$_2$ in mass fraction. The cross sections for electron interactions with N$_2$, O$_2$, and Ar are directly obtained from the LXcat website database [Phelps database, 2014]. For convenience, we show the plotted cross sections in the following sections. BOLSIG+ is used to calculate the electron momentum and energy transfer collision frequencies and ionization rate for the air mixture as a function of electron temperature $U_e$. We compare the calculated collision frequencies and ionization rate to what was previously given in HLO for electron temperatures from $10^{-3}$ eV to 67 eV.

2.2 Electron Interactions with N$_2$, O$_2$, and Ar and Associated Cross Section Analysis

Electrons with an energy range from $10^{-3}$ eV to 10,000 eV interact with N$_2$, O$_2$, and Ar elastically and inelastically. An inelastic collision occurs when the molecules are left in an excited electronic state with a lifetime greater than the relaxation time of the molecular gas. Inelastic reactions can leave the molecule in a rotational, vibrational, or electronically excited state. Electrons can also dissociate the molecule if the excited state is weakly bound or they can produce ionization. Inelastic interactions have a threshold electron energy, which is also the energy lost per inelastic collision. The probability for each of these types of collisional interactions to occur is quantified by the cross section, which is measured in unit area.

For electrons in this energy range, it is common to use an effective momentum transfer cross section, which includes all elastic and inelastic collisions that transfer momentum. This is a convenient parameter to use in the Boltzmann equation. The momentum transfer cross sections, along with the inelastic cross sections described above, are plotted and described in the following sections. For each cross section, the type of cross section, original reference for the cross section data, and last time the cross section data was changed, or updated, in the database is provided. The changes to the cross sections are made so that, for a complete set of cross sections, the
accuracy of swarm and transport parameters calculated with a Boltzmann solver should be within 10% [Pancheshnyi, et al., 2012]. Accuracy is based off of experimental measurements, or a sensitivity test when no experimental data is available. The cross sections shown in the sections below are plotted for the energy range published on the LXcat website in the Phelps database. BOLSIG+ extrapolates cross sections by adding data points beyond the last published energy by increasing $\varepsilon$ by a factor 1.5 for $\varepsilon$ up to 10 keV and assuming that the cross section decreases as $\ln(\varepsilon)/\varepsilon$ [Hagelaar and Pitchford, 2010].

2.2.1 Cross section of electron interaction with N$_2$

The cross sections for N$_2$ are plotted in Figure 1 below.

![Figure 1. Cross sections for electrons interacting with N$_2$ [Phelps Database, 2014].](image_url)
Figure 1 provides information on the energies at which the corresponding cross section is significant. The legend is tabulated in Table 2 below.

Table 2. Cross section reaction and type, reference for original data, and year updated for N₂ [Phelps Database, 2014].

<table>
<thead>
<tr>
<th>Label</th>
<th>Reaction Type</th>
<th>Cross Section Type</th>
<th>Original Reference</th>
<th>Last Updated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Effective</td>
<td>Momentum Transfer</td>
<td>[Phelps and Pitchford, 1985]</td>
<td>2011</td>
</tr>
<tr>
<td>3</td>
<td>Excitation</td>
<td>v1</td>
<td>[Engelhardt et al., 1964]</td>
<td>2009</td>
</tr>
<tr>
<td>4</td>
<td>Excitation</td>
<td>v1 Resonance</td>
<td>[Schulz, 1964]</td>
<td>2010</td>
</tr>
<tr>
<td>5</td>
<td>Excitation</td>
<td>v2</td>
<td>[Schulz, 1964]</td>
<td>2010</td>
</tr>
<tr>
<td>6</td>
<td>Excitation</td>
<td>v3</td>
<td>[Schulz, 1964]</td>
<td>2010</td>
</tr>
<tr>
<td>7</td>
<td>Excitation</td>
<td>v4</td>
<td>[Schulz, 1964]</td>
<td>2010</td>
</tr>
<tr>
<td>8</td>
<td>Excitation</td>
<td>v5</td>
<td>[Schulz, 1964]</td>
<td>2010</td>
</tr>
<tr>
<td>9</td>
<td>Excitation</td>
<td>v6</td>
<td>[Schulz, 1964]</td>
<td>2010</td>
</tr>
<tr>
<td>10</td>
<td>Excitation</td>
<td>v7</td>
<td>[Schulz, 1964]</td>
<td>2010</td>
</tr>
<tr>
<td>11</td>
<td>Excitation</td>
<td>v8</td>
<td>[Schulz, 1964]</td>
<td>2010</td>
</tr>
<tr>
<td>12</td>
<td>Excitation</td>
<td>A³Σ_u⁺ v0-4</td>
<td>[Cartwright et al., 1977]</td>
<td>2009</td>
</tr>
<tr>
<td>13</td>
<td>Excitation</td>
<td>A³Σ_u⁺ v5-9</td>
<td>[Cartwright et al., 1977]</td>
<td>2009</td>
</tr>
<tr>
<td>14</td>
<td>Excitation</td>
<td>B³Π_g</td>
<td>[Cartwright et al., 1977]</td>
<td>2010</td>
</tr>
<tr>
<td>15</td>
<td>Excitation</td>
<td>W³Δ_u</td>
<td>[Cartwright et al., 1977]</td>
<td>2010</td>
</tr>
<tr>
<td>16</td>
<td>Excitation</td>
<td>A³Σ_u⁺ v10-</td>
<td>[Cartwright et al., 1977]</td>
<td>2009</td>
</tr>
<tr>
<td>17</td>
<td>Excitation</td>
<td>B³Σ⁻_u</td>
<td>[Cartwright et al., 1977]</td>
<td>2010</td>
</tr>
<tr>
<td>18</td>
<td>Excitation</td>
<td>a¹Σ⁻_u</td>
<td>[Cartwright et al., 1977]</td>
<td>2010</td>
</tr>
<tr>
<td>19</td>
<td>Excitation</td>
<td>a¹Π_g</td>
<td>[Cartwright et al., 1977]</td>
<td>2010</td>
</tr>
<tr>
<td>20</td>
<td>Excitation</td>
<td>w¹Δ_u</td>
<td>[Cartwright et al., 1977]</td>
<td>2010</td>
</tr>
</tbody>
</table>
The elastic scattering cross section is not plotted in Figure 1 for N₂, or for O₂ and Ar, but is included in the given effective momentum transfer cross section (labeled as 1). The momentum transfer cross section extends over all energies since it is dominated by elastic scattering. Momentum transfer has a peak at around 2 eV, which is due to the large rotational and vibrational resonances around that energy (labeled as 2-11). The low energy tail of the rotational cross section (labeled as 2) extends to the lowest energies of all of the inelastic cross sections. Higher energy electronic excitation cross sections (labeled as 12-23) usually have a maxima at around 10-11 eV but are usually an order of magnitude or more lower than the vibrational cross section. An exception to this is singlet state excitation (labeled as 24) which has a high energy tail that corresponds to a relatively large cross section. The ionization cross section (labeled as 25) is also large at high energies.

2.2.2 Cross section of electron interactions with O₂

The cross sections for O₂ using the Phelps compilation from the LXcat website are plotted below.
Figure 2. Cross sections for electrons interacting with \text{O}_2 [\text{Phelps Database, 2014}].

The descriptions of the cross sections shown in Figure 2 are given in Table 3 below. Similarly to \text{N}_2, the momentum transfer cross section (labeled as 1) for \text{O}_2 is defined for all electron energies. In contrast, there is a depression from 1-6 eV since cross section magnitude (for all inelastic scattering cross sections) is low at these energies. There are sharp rotational and vibrational resonances at low energies (labeled as 2, 3, 5, 7, 8) and wider, 9 eV vibrational resonances (labeled as 4, 6). The 8.4 eV excitation cross section (labeled as 13) has a high energy tail that corresponds to a relatively large cross section, as does ionization (labeled as 15).

Table 3. Cross section reaction and type, reference for original data, and year updated for \text{O}_2 [\text{Phelps Database, 2014}].

<table>
<thead>
<tr>
<th>Label</th>
<th>Reaction Type</th>
<th>Cross Section Type</th>
<th>Original Reference</th>
<th>Last Updated</th>
</tr>
</thead>
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<tr>
<td>No.</td>
<td>Excitation</td>
<td></td>
<td>Reference(s)</td>
<td>Year</td>
</tr>
<tr>
<td>-----</td>
<td>------------</td>
<td>----------------------</td>
<td>-----------------</td>
<td>------</td>
</tr>
<tr>
<td>2</td>
<td>Excitation</td>
<td>Rotational</td>
<td>[Hake and Phelps, 1967]</td>
<td>2010</td>
</tr>
<tr>
<td>3</td>
<td>Excitation</td>
<td>v1</td>
<td>[Linder and Schmidt, 1971]</td>
<td>2010</td>
</tr>
<tr>
<td>4</td>
<td>Excitation</td>
<td>v1 Resonance</td>
<td>[Wong et al., 1973; Trajmar et al., 1972]</td>
<td>2010</td>
</tr>
<tr>
<td>5</td>
<td>Excitation</td>
<td>v2</td>
<td>[Linder and Schmidt, 1971]</td>
<td>2010</td>
</tr>
<tr>
<td>6</td>
<td>Excitation</td>
<td>v2 Resonance</td>
<td>[Wong et al., 1973; Trajmar et al., 1972]</td>
<td>2010</td>
</tr>
<tr>
<td>7</td>
<td>Excitation</td>
<td>v3</td>
<td>[Linder and Schmidt, 1971]</td>
<td>2010</td>
</tr>
<tr>
<td>8</td>
<td>Excitation</td>
<td>v4</td>
<td>[Linder and Schmidt, 1971]</td>
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<tr>
<td>9</td>
<td>Excitation</td>
<td>$a^1\Delta_g$</td>
<td>[Linder and Schmidt, 1971]</td>
<td>2010</td>
</tr>
<tr>
<td>10</td>
<td>Excitation</td>
<td>$b^3\Sigma_g^+$</td>
<td>[Linder and Schmidt, 1971]</td>
<td>2009</td>
</tr>
<tr>
<td>11</td>
<td>Excitation</td>
<td>4.5 eV</td>
<td>[Phelps, Phelps Database, Private Communications]</td>
<td>2010</td>
</tr>
<tr>
<td>12</td>
<td>Excitation</td>
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<td>[Phelps, Phelps Database, Private Communications]</td>
<td>2010</td>
</tr>
<tr>
<td>13</td>
<td>Excitation</td>
<td>8.4 eV</td>
<td>[Chantry and Schulz, 1967]</td>
<td>2010</td>
</tr>
<tr>
<td>14</td>
<td>Excitation</td>
<td>9.97 eV</td>
<td>[Trajmar et al., 1972]</td>
<td>2010</td>
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<tr>
<td>15</td>
<td>Ionization</td>
<td>$O_2^+$</td>
<td>[Phelps, Phelps Database, Private Communications]</td>
<td>2010</td>
</tr>
</tbody>
</table>

The $O_2$ 2- and 3-body attachment cross sections are shown in Figure 3 below.
Figure 3. Attachment cross sections for electrons interacting with O₂ [Phelps Database, 2014]. The three-body attachment cross section is normalized to gas density in units of cm³.

The legend for Figure 3 can be seen in Table 4 below. The three-body attachment (1) is normalized to gas density, and therefore needs to be multiplied by the gas density, in units of cm⁻³ to obtain the cross section. The air density will vary in this analysis since we monitor electron equilibrium for various altitudes.

Table 4. Attachment cross section reaction and type, reference for original data, and year updated for O₂ [Phelps Database, 2014].

<table>
<thead>
<tr>
<th>Label</th>
<th>Reaction Type</th>
<th>Cross Section Type</th>
<th>Original Reference</th>
<th>Last Updated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3-body Attachment</td>
<td>[Phelps, Phelps Database, Private Communications]</td>
<td>2010</td>
<td></td>
</tr>
</tbody>
</table>
2.2.3 Cross section of electron interactions with Ar

The cross sections for Ar using the Phelps compilation from the LXcat website are plotted below.

![Figure 4](image)

**Figure 4. Cross sections for electrons interacting with Ar [Phelps Database, 2014].**

The legend for Figure 4 can be seen in Table 5 below. The momentum transfer cross section for Ar (labeled as 1) spans over all electron energies. There is a depression in the cross section at about 0.2 eV that corresponds to the Ramsauer minimum [Pitchford et al., 2013]. Ar is monatomic so there are no rotational or vibrational excited states; therefore the momentum transfer cross section is based on elastic momentum transfer for electron energies less than 11.5 eV. All of the electronic excitation cross sections (labeled as 2) are grouped together. The total electronic excitation cross section has high energy tail that corresponds to a relatively large cross section, as does ionization (labeled as 3).
Table 5. Cross section reaction and type, reference for original data, and year updated for Ar [Phelps Database, 2014].

<table>
<thead>
<tr>
<th>Label</th>
<th>Reaction Type</th>
<th>Cross Section Type</th>
<th>Original Reference</th>
<th>Last Updated</th>
</tr>
</thead>
<tbody>
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<td>Effective</td>
<td>Momentum Transfer</td>
<td>[Yamabe et al., 1983]</td>
<td>2011</td>
</tr>
<tr>
<td>2</td>
<td>Excitation</td>
<td>All Excitation</td>
<td>[Yamabe et al., 1983]</td>
<td>2010</td>
</tr>
<tr>
<td>3</td>
<td>Ionization</td>
<td>Ar⁺</td>
<td>[Yamabe et al., 1983]</td>
<td>2010</td>
</tr>
</tbody>
</table>

3. RESULTS AND ANALYSIS

In the following sections, we compare the swarm parameters calculated by BOLSIG+, which uses the updated cross sections in Sections 2.2.1-2.2.3 and the methodology in Section 2.1.2, to the rates published in HLO, given in Section 2.1.1. The swarm model given in Section 2.1 employs the BOLSIG+ calculated swarm parameters to calculate electron temperatures at the equilibrium state and the equilibration times at different elevation of atmosphere. The swarm model also employs the rates published in HLO to calculate equilibrium temperatures and times and we compare the results. The swarm model (employing either set of swarm parameters) is validated by comparing to recent experimental data.

3.1 Momentum Transfer Collision Frequency

Figure 5 presents the momentum transfer collision frequency in air for the assumed mixture of 1% Ar, 21% O₂, and 78% N₂ using the BOLSIG+ with the updated momentum transfer cross sections labeled as curve 1 in Figure 1, Figure 2, and Figure 4 in Equation (12). The total momentum transfer collision frequency is a sum of the momentum transfer collision frequency of each component of air times the mass fraction for the respective component (νₓ / N = 0.78(νₓ / N)$_{N_2}$ + 0.22(νₓ / N)$_{O_2}$ + 0.01(νₓ / N)$_{Ar}$). This is compared to previously reported, fitted function given by HLO (Equation (6)).
Figure 5. Momentum transfer collision frequency normalized to atmospheric number density, \( \nu_m / N \), for electrons in air versus characteristic temperature, \( U_e \). Air is assumed to be a mixture of 78% \( \text{N}_2 \), 21% \( \text{O}_2 \), and 1% \( \text{Ar} \) for this analysis. The blue line is the momentum transfer collision frequency found using the HLO equation for \( \nu_m \) and the black line is the momentum transfer collision frequency found using BOLSIG+ to calculate \( \nu_m \).

The momentum transfer collision frequency is consistent between calculations using BOLSIG+ with the updated momentum transfer cross section and the previously reported momentum transfer collision frequency given in HLO. Though, there is an enhancement in the BOLSIG+ calculated collision frequency for characteristic energies from 1.5 to 2.5 eV that is not seen in the HLO results. This is due to effects from the rotational and vibrational resonances in \( \text{N}_2 \) labeled 2-11 in Figure 1. These resonances are large enough to appreciably increase the total momentum transfer to the surrounding air through the inelastic momentum transfer to \( \text{N}_2 \). This causes a noticeable enhancement in the momentum transfer collision frequency calculation for this temperature region. This enhancement is not resolved in the HLO data fits, though the experimental data that HLO is fitting from 1.5 < \( U_e \) < 2.5 eV does show this enhancement [Baum, 1965]. Since the momentum transfer collision frequency is mainly dependent upon the
elastic cross section, we would not expect to see much difference between the previously calculated HLO results since this cross section is well known for the components of air.

### 3.2 Energy Transfer Collision Frequency

Figure 6 presents the energy transfer collision frequency in air found using BOLSIG+, along with up-to-date cross sections, in Equation (13). The energy transfer collision frequency has a very small contribution from elastic energy transfer (see first term in Equation (13)) so the cross sections labeled 1 in Figure 1, Figure 2, and Figure 4 are included in the calculation. Also, the energy transfer collision frequency calculation has a large contribution from the sum of all the inelastic cross sections (see second term in Equation (13)). Therefore, the cross sections used in the calculation include 1-24 in Figure 1 \((\nu_{w} / N)_{N_2}\), 1-14 in Figure 2 \((\nu_{w} / N)_{O_2}\), and 1-2 in Figure 4 \((\nu_{w} / N)_{Ar}\) so that the total energy transfer collision frequency is \((\nu_{w} / N) = 0.78(\nu_{w} / N)_{N_2} + 0.22(\nu_{w} / N)_{O_2} + 0.01(\nu_{w} / N)_{Ar}\). This is compared to the previously reported energy transfer collision frequency given in HLO (Equation (5)).

![Figure 6. Energy transfer collision frequency normalized to atmospheric number density, \(\nu_{w} / N\), for electron in air versus electron characteristic temperature, \(U_e\). Air is assumed to be a mixture of 78% N\(_2\), 21% O\(_2\), and 1% Ar for this analysis. The blue line is the energy](image)

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transfer collision frequency found using the HLO equation for $\nu_w$ and the black line is the energy transfer collision frequency found using BOLSIG+ to calculate $\nu_w$.

The low electron temperature tail ($U_e < 10^{-1}$ eV) for the BOLSIG+ results (black curve) in Figure 6 is due to the low energy tail in the rotational excitation cross section ($\varepsilon < 1$ eV) in N$_2$ labeled 2 in Figure 1. This temperature regime at $\nu_w = 10^{-11}$ cm$^3$/sec is shifted to higher temperatures when compared to the HLO results (blue curve). Although we see this shift at low characteristic temperatures, the BOLSIG+ results give the same general trend as the HLO calculation. Generally, rotational and vibrational excitations from both O$_2$ and N$_2$ transfer energy to the background in the mid-range temperature regime. Higher energy electronic excitations for Ar, O$_2$, and N$_2$, along with the 9 eV vibrational resonances in O$_2$, guide energy transfer in high electron temperature regimes. As stated in Section 2.2.1, the HLO results are from fit to the best available data (with direct experimental measurements for $U_e < 3$ eV and calculations using scattering cross sections for $U_e > 3$ eV) in 1973. Differences in the BOLSIG+ curve and HLO curve may be attributed to the BOLSIG+ data being calculated with a BOLSIG+ EEDF and the updated cross section data set over the entire temperature range in Figure 6. Updates in the cross section compilation are based upon new experimental data and further testing and validation with transport parameters. It should be noted that the energy transfer collision frequency in HLO will increase unphysically with increasing electron temperature since it is only meant to be valid below about 67 eV.

3.3 Ionization Rate

The ionization rate presented in HLO is given in Equation (7). The general form of the ionization rate, shown by Equation (8), is used to find the ionization rate with BOLSIG+ using updated cross sections. These results are compared in Figure 7 below.
Figure 7. Ionization rate normalized to atmospheric number density, $C_i/N$, for electron in air versus electron characteristic temperature, $U_e$. Air is assumed to be a mixture of 78% N$_2$, 21% O$_2$, and 1% Ar for this analysis. The blue curve is the ionization rate found using the HLO equation for $C_i$ and the black curve is the ionization rate found using BOLSIG+ to calculate $C_i$.

The ionization rate calculated using BOLSIG+ is smaller than the ionization rate given in HLO, with the exception of characteristic energies around 20 eV, and can be up to an order of magnitude smaller for electrons with a few eV characteristic energies. It is important to note that there was no data for the HLO ionization rate fit for $3 \text{ eV} < U_e < 20 \text{ eV}$ and is not meant to be valid above 67 eV.

3.4 Validation of Swarm Model with Experimental Data

Validation is necessary to demonstrate that the swarm model will be able to accurately calculate transport parameters for both an equilibrium and non-equilibrium scenario. Experimental measurements are reported in the literature as function of the reduced electric field, $E/N$. Steady state swarm model calculations allow for a mapping of the characteristic
temperature and electric field, as can be seen by referring to Equation 2. These results are presented in Section 3.4.1 below. Through this relationship, swarm parameters and transport coefficients can be evaluated as a function of $E / N$ and compared to experiments. For validation, we compare the swarm calculated effective ionization coefficient in air to experimental results provided in [Dutton, 1975] and drift velocities to experimental data given in [Ruiz-Vargas et al., 2010]. The effective ionization coefficient and electron drift velocity are important parameters for our validation effort because they will be needed in conductivity, and therefore conduction current, calculations. The ionization coefficient and drift velocity validation results are given in Section 3.4.2 and 3.4.3, respectively.

3.4.1 Reduced Electric Field and Characteristic Energy Mapping

In order to map $E / N$ to $U_e$, we impose certain initial conditions and allow the electron swarm to reach steady state. An equilibrium characteristic temperature $U_e$ can then be found for an imposed reduced electric field $E / N$ that is based on the swarm electrons equilibrium distribution. We use the BOLSIG+ calculated swarm parameters, as well as the HLO swarm parameters, in the swarm model to calculate the equilibrium $U_e$. It has been demonstrated previously [e.g., Ridenti et al., 2010; Peng-Cheng et al., 2014] that BOLSIG+ is a high-fidelity Boltzmann solver within a range of physical conditions that extend to our problem description.

For the swarm model calculation, the electric field imposed is $1 \text{ StatV/cm}$. The electric field is allowed to evolve in the swarm model, but remains constant because the secondary number density stays small. The initial electron temperature is $5 \text{ eV}$ since, as stated previously, we assume secondaries start with an energy of about $8 \text{ eV}$. The number density of the electrons is arbitrarily chosen to be $1 \text{ cm}^{-3}$. The initial drift velocity is calculated based off the relationship

$$v_d = \frac{\mu}{E} = \frac{e}{m_e \nu_m(U_e) E},$$

where $\nu_m(U_e)$ is evaluated at $U_e = 5 \text{ eV}$ and $E$ is the imposed electric field. The number density for air molecules will vary with atmospheric height, $H$, in km. We assume $N = N_0 \exp(-H/7)$ [Higgins et al., 1973] ($N_0$ is the number density of air at sea level, $2.97 \times 10^{25} \text{#/cm}^3$) for this analysis. The electric field is kept constant and we vary the air number density to obtain a reduced electric field range of $1 \text{Td} < E / N < 1,000 \text{Td}$. The results are shown in Figure 8 below.
Figure 8. Electron temperature, $U_e$, versus reduced electric field, $E/N$. The red line is the equilibrium $U_e$ calculated by the swarm model using the swarm parameters given in HLO and the blue line is the equilibrium $U_e$ calculated by the swarm model using the BOLSIG+ calculated swarm parameters.

The relationship between $E/N$ and $U_e$ provided in Figure 8 allows us to remap $\nu_m$, $\nu_w$, and $C_1$ shown in Figure 5-Figure 7 and compare various parameters, including the ionization coefficient, to experiments. We also use the same procedure described above, as well as the same initial conditions, to calculate transport parameters, including the drift velocity, as a function of $E/N$ and compare to experiments. These two studies are performed in the following sections for model validation.

### 3.4.3 Swarm Electron Effective Ionization Rate Data Validation

The effective ionization coefficient is calculated by computing the ionization rate less the attachment rate over the drift velocity at a specific $E/N$ ($\dot{\alpha}(E/N) = \frac{C_1(E/N) - \alpha(E/N)}{v_d(E/N)}$). We use the remapping shown in Figure 8 to obtain $C_1(E/N)$ and $\alpha(E/N)$ and find the steady state
drift velocity \( v_d(E/N) \) by employing the same initial conditions given above and allowing the electrons to reach their equilibrium distribution. The 3-body attachment rate is determined at \( H = 0 \) km. Results are obtained for a reduced electric field \( E/N \) range from \( 150 \text{ Td} < E/N < 1,000 \text{ Td} \) and are compared to the experimental values in air from the [Dutton, 1975] data compilation. The experimental data was found using the steady-state Townsend technique which includes finding the ionization coefficient by monitoring the ionization current through an electrode [Rao and Raju, 1971]. The results are shown in Figure 9 below.

![Figure 9](image)

**Figure 9.** Effective ionization coefficient, \( \lambda \), versus reduced electric field, \( E/N \). The black x’s represent the experimental data from [Dutton, 1975], the red line is the equilibrium \( \lambda \) calculated by the swarm model using the swarm parameters given in HLO and the blue line is the equilibrium \( \lambda \) calculated by the swarm model using the BOLSIG+ calculated swarm parameters.

Figure 9 indicates that the swarm model calculated effective ionization coefficient is in good agreement with experiments. This is true when utilizing either the HLO or BOLSIG+ calculated swarm parameters. The experimental results for \( E/N < 280 \text{ Td} \) have better agreement with the effective ionization coefficient calculated using the HLO coefficients. For \( 280 \text{ Td} < \)
$E/N < 450$ Td, the effective ionization coefficient calculated using the BOLSIG+ coefficients tends to have better agreement with experiments. Since the experimental data is scattered for $E/N > 450$ Td it is difficult to determine which set of swarm parameters produces the best results but it is clear that both sets facilitate good agreement.

3.4.3 Swarm Electron Drift Velocity Data Validation

Experimentally measured values of drift velocity in air are given in [Ruiz-Vargas et al., 2010]. In this work, the drift velocity measurements were carried out over a range of pressures, corresponding to a reduced electric field $E/N$ range from $1$ Td $< E/N < 1,000$ Td (1 Td $= 10^{-17}$ V cm$^2 = 3.33 \times 10^{-17}$ statV cm$^2$) using pulsed Townsend techniques with overall uncertainties ranging from 1-1.5%. We use the BOLSIG+ calculated swarm parameters, as well as the HLO swarm parameters, in the swarm model to calculate equilibrium drift velocities and compare to the experimental drift velocities from [Ruiz-Vargas et al., 2010].

For the swarm model calculation, we impose initial conditions and calculate the drift velocity when the electrons reach their equilibrium distribution. The experimental results and the results of the swarm calculation are shown in Figure 10 [Error! Reference source not found.].
Figure 10. Electron drift velocity, \( v_d \), versus reduced electric field, \( E/N \). The black x’s represent the experimental data from [Ruiz-Vargas et al., 2010], the red line is the equilibrium \( v_d \) calculated by the swarm model using the swarm parameters given in HLO and the blue line is the equilibrium \( v_d \) calculated by the swarm model using the BOLSIG+ calculated swarm parameters.

Figure 10 indicates that the swarm model calculated drift velocities are in good agreement with experiments. This is true when utilizing either the HLO and BOLSIG+ calculated swarm parameters, though, when using the BOLSIG+ swarm parameters, the calculated drift velocities better represent the experimental results for a wider range of \( E/N \). As well as being part of the validation effort, Figure 10 provides some insight into the sensitivity of swarm calculations to the swarm parameters used.

3.5 Electron Equilibration Time and Temperatures

We study the equilibration temperature and time as a means of quantifying how important it is to allow the conduction electrons to evolve in time for various EMP altitudes. If
the equilibration timescale is on the order of or greater than the timescale of the EMP for a specific altitude, then it is possible that the electrons will not have enough time to reach equilibrium, and the non-equilibrium model should be used. Even if the time to reach equilibrium is short with respect to the EMP duration or rise time and the equilibrium transport parameters can be used, our validation efforts in Section 3.4 show we are able to calculate physically realistic swarm and transport parameters, which will give accurate values of conductivity and conduction current.

Equation (2) shows the time derivative of electron temperature. The equilibrium electron temperature $U_{eq}$ will be higher than thermal in the presence of an electric field. Conduction electrons will eventually reach an equilibrium distribution when the energy gained by the electrons from the electric field is balanced by the energy loss through collisions with the surrounding gas [Longmire and Longley, 1973]. As can be seen in Figure 7 the ionization rate $C_i$ is very sensitive to changes in the electron temperature. This is especially true when approaching the ionization threshold, where the ionization rate can increase or decrease over an order of magnitude with 1 eV change in temperature. The same initial conditions presented in Section 3.4.1-3.4.3 are imposed for the remaining calculations. As before, since the electric field remains constant, the equilibrium temperature is not sensitive to the initial temperature because the applied electric field, the atmospheric density, and the set of swarm parameters used control the equilibrium distribution. This is not necessarily true for the equilibration time since the collision rate will increase or decrease depending on the initial electron temperature, so the time to reach the equilibrium distribution would be affected.

The equilibrium temperatures calculated when employing the HLO swarm parameters and calculated using BOLSIG+ with the updated cross sections to evaluate swarm parameters are shown in Table 6 below for $H =$0 km, 10 km, 20 km, 30 km, and 40 km.

<table>
<thead>
<tr>
<th>$H$ [km]</th>
<th>$U_{eq}$ (HLO) [eV]</th>
<th>$U_{eq}$ (BOLSIG+) [eV]</th>
</tr>
</thead>
</table>

**Table 6. Equilibrium temperature, $U_{eq}$, versus atmospheric height, $H$, found from the swarm model using the HLO swarm parameters ($U_{eq}$ (HLO)) and BOLSIG+ with the updated cross sections to calculate swarm parameters ($U_{eq}$ (BOLSIG+)).**
Generally, the equilibrium temperatures calculated using the HLO and BOLSIG+ swarm parameters in the swarm model are in good agreement with one another but there are differences that can be attributed to differences in the calculated collision frequencies and ionization rate. For example, in regions where the BOLSIG+ energy transfer collision frequency is lower than the HLO energy transfer collision frequency (see Figure 6), the equilibrium temperature will be higher because less energy is being transferred to the background air molecules and vice versa. In this analysis, the number density of air decreases with increasing altitude so the equilibrium temperature will increase since there are less electron-molecule interactions. The BOLSIG+ energy transfer collision frequency is higher than the HLO energy transfer collision frequency for $1.3 \text{ eV} < U_e < 1.8 \text{ eV}$ which explains why the equilibrium temperature at 30 km using the HLO rates is higher than when using the BOLSIG+ rates. As shown in Figure 11 below, electron attachment will be low relative to ionization for the initial condition on $U_e$. This, along with the fact that $\frac{dN_e(t)}{dt}$ remains low, means that attachment does not have a significant effect on our calculations.
Figure 11. Attachment rate normalized to atmospheric number density, $\alpha / N$, for electron in air versus electron characteristic temperature, $U_e$. Air is assumed to be a mixture of 78% N$_2$, 21% O$_2$, and 1% Ar for this analysis. The curves represent the attachment rate found using the updated cross sections and BOLSIG+ to calculate $\alpha$. Since 3-body attachment is normalized to gas density, the attachment rate is plotted for H=0-40 km.

The 3-body attachment cross section, which is labeled as 1 in Figure 3, is normalized to gas density so the attachment coefficient is calculated for H=0-40 km. We can see that attachment will be greater at lower atmospheric heights where the number density of oxygen is higher. Attachment may have a greater effect on our calculations under different physical conditions and, for example, if a source of conduction electrons is introduced.

The time it takes to reach the equilibrium temperature, $t_{eq}$, is shown in Table 7.

Table 7. Equilibration times, $t_{eq}$, versus atmospheric height, $H$, found from the swarm model using the HLO swarm parameters ($t_{eq}$ (HLO)) and BOLSIG+ with the updated cross sections to calculate swarm parameters ($t_{eq}$ (BOLSIG+)).
<table>
<thead>
<tr>
<th>( H ) [km]</th>
<th>( t_{eq} ) (HLO) [s]</th>
<th>( t_{eq} ) (BOLSIG+) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.32E-09</td>
<td>1.27E-09</td>
</tr>
<tr>
<td>10</td>
<td>3.75E-09</td>
<td>3.27E-09</td>
</tr>
<tr>
<td>20</td>
<td>1.00E-9</td>
<td>7.76E-10</td>
</tr>
<tr>
<td>30</td>
<td>8.52E-10</td>
<td>1.15E-09</td>
</tr>
<tr>
<td>40</td>
<td>1.13E-09</td>
<td>8.74E-10</td>
</tr>
</tbody>
</table>

By comparing equilibration times in Table 7 at a given atmospheric height, and considering the corresponding equilibrium temperatures in Table 6, we can see that the electrons will take less time to equilibrate to higher temperatures and more time to equilibrate to lower temperatures. This is true specifically for our initial conditions given, where the electrons lose energy with respect to their initial energy or temperature, as time progresses.

The equilibration times were found at 99% equilibrium for the convergence criteria. This can be seen in Figure 12 below. 90% equilibrium is also shown on this plot.

![Figure 12](https://via.placeholder.com/150)

**Figure 12.** (a) Characteristic temperature as a function of time at an altitude of 30 km when running the swarm model with the rate coefficients from the HLO paper. (b) Characteristic temperature as a function of time at an altitude of 30 km when running the swarm model with the updated BOLSIG+ rate coefficients. The grey line represents the equilibration time at 90% of the equilibrium temperature (4.86E-10 s and 6.91E-10 s,
respectively), and the red line represents the equilibration time at 99% of the equilibrium temperature (8.52E-10 s and 1.15E-09 s, respectively).

Figure 12 demonstrates the importance of monitoring the conduction electron equilibration. Only reaching 90% equilibrium from \( U_e = 5 \) eV can produce a different equilibration time (and temperature) than if we let the electrons reach 99% equilibrium from \( U_e = 5 \) eV. These two times differ because the collision frequencies and ionization rate are changing at each time step as the conduction electrons are evolving. This is a fundamental reason why we study electron equilibration. As can be seen when referring to Figure 6, when the electron energy is high, the rate of energy change is high. So, it takes very little time to go from a high energy to near-equilibrium energy, but much longer to go from near-equilibrium energy to the equilibrium energy. Therefore, the rate of cooling is slower closer to the equilibrium temperature. This effect can also be seen in Figure 12. This means that, for the initial conditions we impose, increasing the initial temperature will not significantly change the equilibration time we have reported. The same effect occurs due to ionization, as seen in Figure 7, but is even more pronounced since energy loss due to ionization is dominant at high temperatures drops rapidly to zero at temperatures below the ionization threshold. However, this effect is only significant at high temperatures. Figure 12 also indicates that, in general, convergence criteria should be chosen carefully when implementing the swarm model. The adaptive time step is useful in this model since it can conserve computational time by adjusting the time step necessary for the calculation.

4. CONCLUSION AND FUTURE WORK

The calculated results for the energy and momentum transfer collision frequencies and ionization rate using BOLSIG+ are compared to fits given in the HLO paper designed for use in an electron swarm model. Validation of the swarm model with experimental results for the effective ionization coefficient and electron drift velocity in air for a range of reduced electric fields is presented. The sensitivity of the equilibrium temperature and time calculation on updating the swarm model parameters is studied and the swarm calculated equilibrium temperatures are compared to experimental data. Since the equilibrium temperature and time calculations were indeed sensitive to the changes in collision frequencies and ionization rate, we
conclude that different cross section sets can change the results obtained from the swarm model. Using the most up-to-date and complete data is important and recommended based on our validation efforts. Careful analysis on the cross section data has been done on the LXcat website which can be used as a tool for modeling low energy electron physics. Testing the sensitivity of the swarm parameters to different sets of cross sections in the LXcat database is warranted as future work.

Other future work lies in including water vapor content and other common components of air, such as CO$_2$, to the swarm model. Additional physical processes, such as recombination and mutual neutralization, will also be included in the swarm model to represent the most realistic physical configuration. Including a physically realistic production rate of the low energy electrons $S_1$ and testing swarm model sensitivity to $S_1$ also warrant further investigation. After validation of the swarm model with the additional physical parameters included, the conductivity calculated by the swarm model can be used in the solution the Maxwell’s equations to model EMP.

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