Parameter Sensitivity Study on the Accuracy of Chord Length Sampling Method

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INTRODUCTION

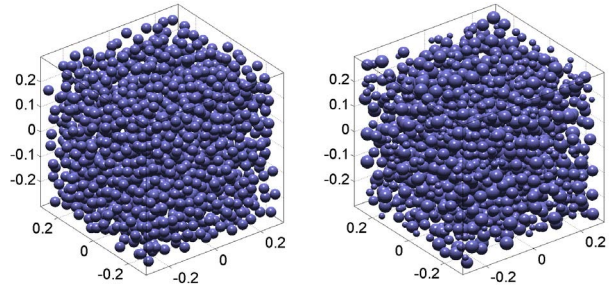
Chord length sampling (CLS) method has been studied for a long time since first proposed [1]. Much work has been done on the CLS efficiency and accuracy study to solve radiation transport problems in 1D/2D/3D geometry, where fixed-size inclusions are randomly distributed [2]-[5] or two random-size inclusions are alternately distributed [6]. Current major application of CLS in nuclear reactor systems is to analyze the stochastic distribution of fuel particles in the Very High Temperature Gas-cooled Reactors (VHTRs), either prismatic type or pebble bed type [7]-[9]. In practice, the accuracy of CLS is problem-dependent, which poses dilemma for one to use CLS. To provide pragmatic guidance in the application of CLS, a systematic investigation of the impacts of problem-dependent parameters, such as material packing fraction, cross section, and system size on the accuracy of CLS is needed. Such an investigation can help relieve the dilemma and give a range of scenarios in which CLS is accurate and can be applied to.

A literature review has shown that investigations have been done for fixed-source problems. Donovan and Danon [3] applied CLS (The LCLS algorithm in the paper) to solving for reflection/leakage rates in a 2D rectangular region composed of disk inclusions. It was found that CLS method gives poorer solutions as the scattering cross section increases in the background region. Reinert et al. [10] has studied the accuracy of CLS by solving a point source problem in a spherical region containing randomly dispersed spherical fuel kernels. By calculating the leakage rate, the relative errors were obtained as a function of three parameters: the ratio of the mean chord length $<l_b>$ to the mean free path λ_b in background material; the ratio of the mean chord length $<l_{fk}>$ to the mean free path λ_{fk} in fuel kernels; and the ratio of $<l_b>$ to $<l_{fk}>$. It concluded that CLS deteriorates most significantly with highly scattering background material, and very dark densely packed fuel kernels. Recently, Brantley [6] systematically investigated the accuracy of two CLS algorithms in 1D fixed-source problems by comparing not only the reflection/leakage rates but the spatial flux distributions. Similar conclusions as Reinert et al. have been drawn though the random media system Brantley studied was constructed in a different way.

In the applications of CLS to analyze nuclear reactor systems, such as VHTRs, eigenvalue problems are more important in the reactor analysis including steady-state and fuel depletion calculations. In this summary, parameter sensitivity study is performed for eigenvalue problems in a cube of scattering background medium, where monosized or polysized fuel kernels are randomly distributed. Multiplication factor error analysis between reference and CLS simulations is done at the variation of a few selected parameters for a range of packing fractions.

RADIATION TRANSPORT PROBLEM AND METHODOLOGY DESCRIPTION

Figure 1 shows two systems that are studied. The radius of each fuel kernel is 0.0185cm in the monosized sphere system, and uniformly distributed over [0.010, 0.025] (cm) in the polysized sphere system.



(a) Monosized fuel kernels (b) Polysized fuel kernels

Figure 1. Cubic systems consisting of monosized or polysized fuel kernels

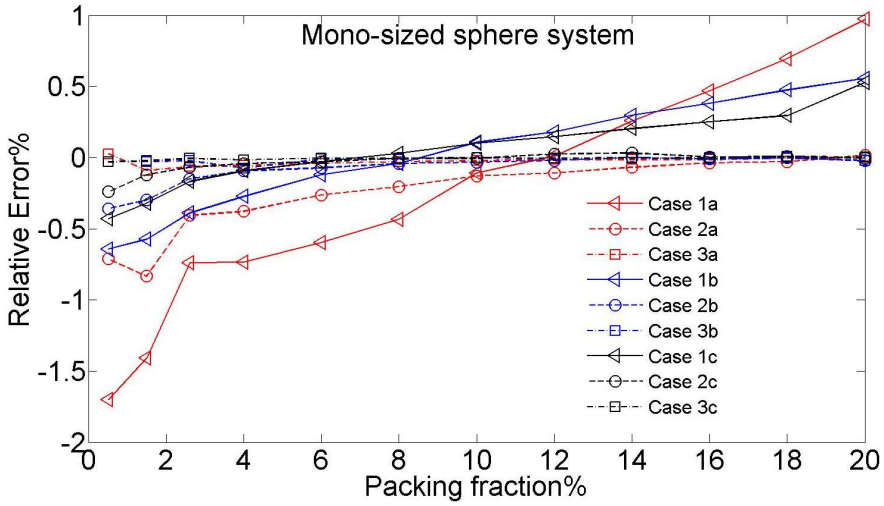
In the investigation, a series of eigenvalue problems are simulated by adjusting fuel kernel cross sections and system sizes at various volume packing fractions. A list of the selected parameter values is shown in Table I. Cases 1 to 3 represent three sets of fuel kernel cross sections that have the ratio $<l_{fk}>/\lambda_{fk}$ about 0.1, 1.0 and 10. These correspond to the scenarios that neutron has rare, a few, and many interactions within the fuel kernel. Cases a to c represent three sets of system sizes. A total of combined 9 cases are studied at the fuel kernel volume packing fractions ranging from 0.52% to 20.0%.

The CLS method is performed within the background medium based on a theoretically derived exponential distribution function [4]. Neutrons are tracked based on regular Monte Carlo procedure within fuel kernels that are forgotten when the neutron exits the kernel [5]. In the CLS simulation, a total of 1 million histories are tracked in each fission generation, with 400 inactive generations over 500 total generations. In the reference case simulation, a fast Random Sequential Addition (RSA) method [11] is used to pack fuel kernels in the cubic system. Same history and batch numbers as the CLS simulation are used for each physical realization. The ensemble-averaged solutions are obtained over 100 realizations. Collision estimator is used for both reference case and CLS simulations.

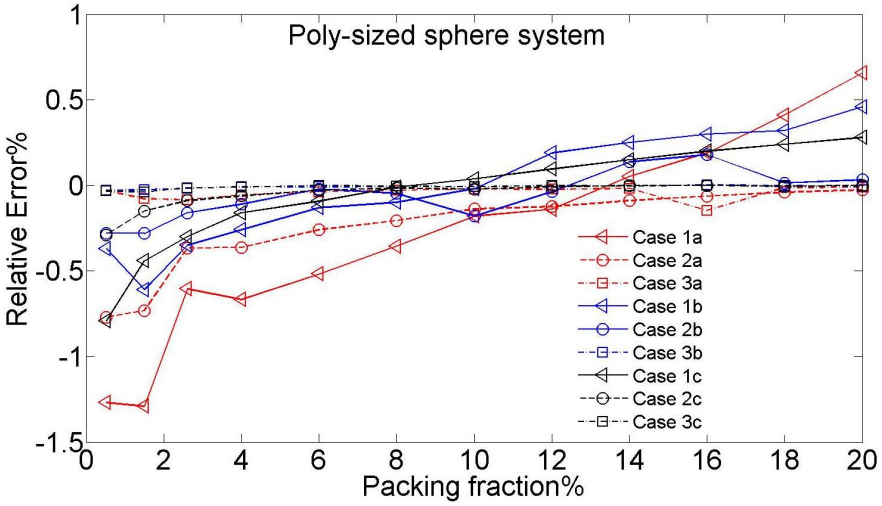
TABLE I Material and geometry parameters in reference case and CLS simulations

Case	$\Sigma_{t,b}$	$\Sigma_{a,b}$	$\Sigma_{s,b}$	$\Sigma_{t,fk}$	$\Sigma_{a,fk}$	$\Sigma_{f,fk}$	$\nu\Sigma_{f,fk}$	$\Sigma_{s,fk}$	$<l_{fk}>/\lambda_{fk}$	Case	System size
1	0.4137	0	0.4137	4.0	2.0	0.8	2.0	2.0	0.1	a	4x4x4 cm
2	0.4137	0	0.4137	40	20	8	20	20	1.0	b	8x8x8 cm
3	0.4137	0	0.4137	400	200	80	200	200	10	c	12x12x12 cm

Note: The subscript "b" denotes background material, "fk" denotes fuel kernel. The unit of cross section data is [cm^{-1}].



(a) Relative errors in monosized sphere system



(b) Relative errors in polysized sphere system

Figure 2. Comparison between CLS and reference case as a function of problem-dependent parameters

RESULTS AND ANALYSIS

Figure 2 shows the relative errors of the predicted multiplication factors between CLS and reference case simulations. For each predicted value, the standard deviation is within 0.01%. It can be seen that the relative errors show quite similar characteristics for both monosized and polysized sphere systems.

Firstly, at the same system size and fuel volume packing fraction, the relative errors decrease consistently as the fuel kernel total cross section increases. This phenomenon can be explained by relating it to the boundary effects [7]-[9]. Due to not allowing overlapping with the external boundary in the CLS, the total fuel kernel inventory would be less than the reference case. Depending on which reaction dominates in the fuel kernel, fission, gamma capture or scattering, the predicted multiplication factors in CLS may deviate from the reference case in a negative or positive sense. When the fuel kernel becomes darker, i.e. the total cross section increases, this deviation would be mitigated and smaller.

Secondly, at the same fuel kernel cross section and volume packing fraction, the relative errors decrease consistently as the system size increases. This can also be explained by the boundary effect as mentioned above. As the system size increases, the loss of the fuel kernel inventory in the CLS relative to the inventory in the reference case would be smaller considering that the total number of fuel kernels increases much faster. Again, depending on the relative significance of the interactions that happen in the fuel kernel, the smaller inventory loss makes the multiplication factor deviation smaller.

Finally, at the same system size and fuel kernel cross section, the relative error shows an anti-symmetric trend for the studied range of packing fractions. When the packing fraction is less than about 10%, k_{eff} values from CLS are less than the ones from reference case. When the packing fraction is higher than 10%, the k_{eff} values are greater than the reference case. This can be explained by two major factors. One is the boundary effect. As the volume packing fraction increases, the total number of the fuel kernels increases. As a result, the loss of the fuel kernel in CLS relative to the total inventory of fuel kernel in the reference case becomes smaller. So the relative error is smaller. However, another factor will have an opposite effect on the relative error at the same time. This second factor is due to the chord length distribution function used in the CLS. A theoretical exponential function $f(l)$ is used in the CLS simulation. Previous investigation has shown [12] that $f(l)$ is very close to the actual chord length distribution $f_{b\to fk}(l)$ from background to fuel kernel but is only an approximation to the actual chord length distribution $f_{fk\to fk}(l)$ between two fuel kernels. At the low volume packing fraction, fuel kernels are loosely distributed and the mean chord length between them $<l_b>$ is larger than the mean free path in the background λ_b. This results in more scatterings in the background and CLS is more frequently performed from background to fuel kernels. Since $f(l)$ is close to $f_{b\to fk}(l)$, the relative error of CLS from the reference case is small. As the packing fraction increases, $<l_b>$ becomes smaller than λ_b. More neutrons will jump from one fuel kernel to another without interactions in the background. Hence CLS is more frequently performed from fuel kernel to fuel kernel. Since $f(l)$ is less close to $f_{fk\to fk}(l)$ than to $f_{b\to fk}(l)$, the relative error becomes large. The combined effect from these two factors is that at some point the relative error becomes zero as shown in Fig. 2 if one factor causes negative errors and the other causes positive errors.

CONCLUSIONS

The accuracy of the CLS method is investigated in eigenvalue problems at the variation of a few selected problem-dependent parameters. It is found that as the system size and fuel kernel cross section increase, the accuracy in k_{eff} values predicted by the CLS method increases. As the fuel kernel volume packing fraction increases, a zero relative error may exist for the k_{eff} values obtained from the CLS method. This conclusion holds at least for the case studies in this summary.

Future sensitivity studies will be performed at higher fuel volume packing fractions up to 30%, different background material cross sections and different system geometry, such as sphere or cylinder. Except multiplication factors, fission source and flux distribution will be calculated for the parameter sensitivity study.

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